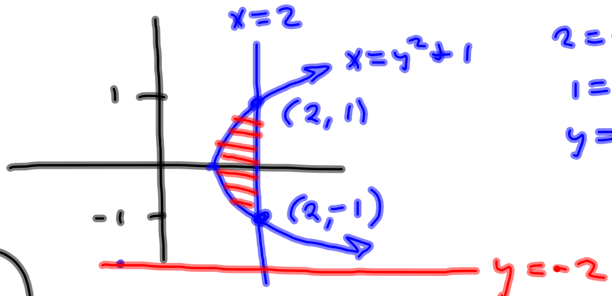
§5.3 #20

volume of solid of revolution obtained when $x=y^2+1$, $x=2$ region is rotated about $y=-2$

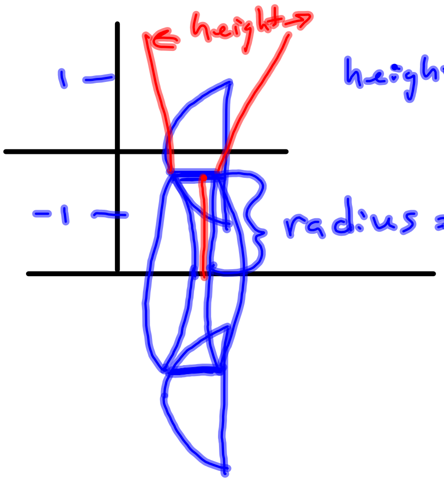


$2=y^2+1$

$1=y^2$

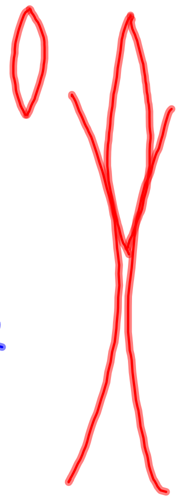
$y=\pm 1$

M1 Shell Method $\quad 2\pi\int_a^b y\,f(y)\,dy$



$\text{height} = 2-(y^2+1)$

$\text{radius} = y-(-2) = y+2$



$$2\pi \int_{-1}^{1} (y+2)(2-y^2-1)\,dy$$

$$= 2\pi \int_{-1}^{1} (y+2)(1-y^2)\,dy$$

$$= 2\pi \int_{-1}^{1} (y-y^3+2-2y^2)\,dy = 2\pi \left[\underbrace{\int_{-1}^{1} (y-y^3)\,dy}_{\to\, 0} + \int_{-1}^{1} (2-2y^2)\,dy \right]$$

(odd fn symmetric interval)

$$= 2\pi \int_{-1}^{1} (2-2y^2)\,dy = 4\pi \int_{-1}^{1} (1-y^2)\,dy$$

$$= 8\pi \int_{0}^{1} (1-y^2)\,dy$$ (Even function Symmetric Interval)

Symmetry is useful!

$$= 8\pi \left[y - \tfrac{1}{3}y^3 \right]_0^1 = 8\pi \left[1 - \tfrac{1}{3} \right] = \tfrac{2}{3} \cdot 8\pi = \frac{16\pi}{3}$$

Solve :

$2\cos^2 x + \cos x - 1 = 0$ } Homework

① $2\csc^2 x + \csc x - 1 = 0$

$\left.\begin{array}{l} \cos^2 x - \sin^2 x = 0 \\ \tan^2 x - 3 = 0 \\ \cot^2 x - 3 = 0 \end{array}\right\}$ Homework

$2\sec^2 x + \sec x - 1 = 0$

find $\frac{dy}{dx}$:

$x^2y^3 - 5x\cos y = 2xy + 5\sin x$ } Homework

② $x^3y^2 - 4y\sec y = 3x^2y - 5\cos x$

1. $2\csc^2 x + \csc x - 1 = 0$

$2u^2 + u - 1 = 0$

$a = 2,\ b = 1,\ c = -1$

$b^2 - 4ac = 1^2 - 4(2)(-1)$

$= 9$

$\sqrt{9} = 3$

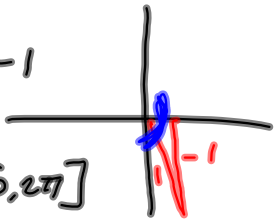
$u = \dfrac{-1 \pm 3}{2(2)}$ → $\dfrac{2}{4} = \dfrac{1}{2}$; $-\dfrac{4}{4} = -1$

$u = \csc x = \frac{1}{2}$
Nevah!

$u = \csc x = -1$

$u = \frac{3\pi}{2}$

Answers on $[0, 2\pi]$



$\csc x = -1$

$\frac{1}{\sin x} = -1$

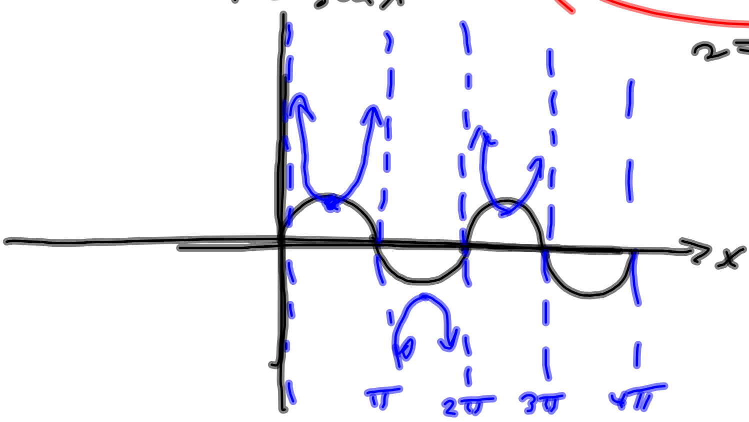
$1 = -\sin x$

$-1 = \sin x$

$\csc x = \frac{1}{2}$

$\frac{1}{\sin x} = \frac{1}{2}$

$2 = \sin x$ Nevah!



$\frac{d}{dx}$ $(fg)' = f'g + fg'$

(2) $x^3y^2 - 4y\sec y = 3x^2y - 5\cos x$

fg Implicit Differentiation

$f = x^3 \quad f' = 3x^2$

$g = y^2 \quad g' = 2yy'$

$$\frac{d}{dx}\left[x^3y^2\right] = \underset{f'g}{3x^2y^2} + \underset{f\,g'}{x^3(2yy')}$$

$$3x^2y^2 + 2x^3yy' - 4y'\sec y - 4y(\sec y\tan y)y' = 6xy + 3x^2y' + 5\sin x$$

$$2x^3yy' - 4y'\sec y - 4y(\sec y\tan y)y' - 3x^2y' = -3x^2y^2 + 6xy + 5\sin x$$

$$y'(2x^3y - 4\sec y - 4y\sec y\tan y - 3x^2) = -3x^2y^2 + 6xy + 5\sin x$$

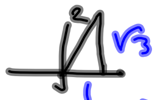
$$\boxed{y' = \frac{-3x^2y^2 + 6xy + 5\sin x}{2x^3y - 4\sec y - 4y\sec y\tan y - 3x^2}}$$

Find equation of tangent line to

$f(x) = \cos x$ @ $x = 60^\circ = \frac{\pi}{3}$ radians.

$f'(x) = -\sin x$ $\quad (60^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3}$



$= \frac{1}{2} \rightsquigarrow \left(\frac{\pi}{3}, \frac{1}{2}\right) = (x_1, y_1)$

$f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} = m$

$y = m(x - x_1) + y_1$

$= -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) + \frac{1}{2}$

Estimate $\cos(65^\circ)$ this way.

$65^\circ = 60^\circ + 5^\circ = \frac{\pi}{3} + 5^\circ = \frac{\pi}{3} + \frac{5\pi}{180}$

$= \frac{\pi}{3} + \frac{\pi}{36}$ radians

$\cos(65^\circ)$

$\approx y(65^\circ) = -\frac{\sqrt{3}}{2}\left(\frac{\pi}{3} + \underbrace{\frac{\pi}{36}}_{\Delta x} - \frac{\pi}{3}\right) + \frac{1}{2}$

$= -\frac{\sqrt{3}}{2}\left(\frac{\pi}{36}\right) + \frac{1}{2}$

$= \frac{-\sqrt{3}\pi + 36}{2}$

Find $\frac{dy}{dx}$

$$x^2y^3 + \sin x \cos(y^2) = 3xy^2$$

$$2xy^3 + x^2(3y^2y') + \cos x \cos(y^2) + \sin x\left(-\sin(y^2)(2yy')\right)$$
$$= 3y^2 + 3x(2yy')$$

$$2xy^3 + 3x^2y^2y' + \cos x \cos(y^2) - 2yy'\sin x \sin(y^2)$$
$$= 3y^2 + 6xyy'$$

$$3x^2y^2y' - 2yy'\sin x \sin(y^2) - 6xyy' =$$
$$= -2xy^3 - \cos x \cos(y^2) + 3y^2$$

$$y'\left(3x^2y^2 - 2y\sin x \sin(y^2) - 6xy\right) = -2xy^3 - \cos x \cos(y^2) + 3y^2$$

$$y' = \frac{-2xy^3 - \cos x \cos(y^2) + 3y^2}{3x^2y^2 - 2y\sin x \sin(y^2) - 6xy}$$

Major Theorems

IVT

MVT for derivatives

MVT for integrals

FTC I

FTC II

Newton's Derivation.
Derivative by Def'n (Polynomial)
Set up Riemann Sum for Def'n of Integral
Left endpt $= 0 = a$
Bonus " " $\neq 0$

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2 or 1 Big Graphing prob on Take-home

Simpler graphing Prob or 2
where you find max/min & IP's
& give rough sketch.
Polynomial & "simple" rational function.

Derivatives & Antiderivatives.
Substitution Rule < Definite / Indefinite
Area between 2 curves
Volumes of solids of revolution } Setup integral is main thing.
Disk
Shell
→ Tests your algebra skills.

1 (a) Write Riemann Sum for $\int_0^3 (x^2-2x)\,dx$
n rectangles right endpts.

(b) Same for $\int_1^3 (x^2-2x)\,dx$

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2 Find $c \ni f(c) = f_{AVG}$ for $f(x) = x^2 - 2x$ on $[0,3]$

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1 a $\Delta x = \frac{b-a}{n} = \frac{3}{n}$

$x_K = a + \Delta x K = 0 + \frac{3}{n}K = \frac{3K}{n}$

$$\sum_{K=1}^{n} f(x_K)\Delta x = \sum_{K=1}^{n} (x_K^2 - 2x_K)\Delta x$$

$$= \sum_{K=1}^{n} \left[\left(\left(\frac{3K}{n}\right)^2 - 2\left(\frac{3K}{n}\right)\right)\left(\frac{3}{n}\right)\right]$$

1 b $\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$x_K = a + \frac{2}{n}K = 1 + \frac{2K}{n}$

$$\sum_{K=1}^{n} \left[\left(\left(1 + \frac{2K}{n}\right)^2 - 2\left(1 + \frac{2K}{n}\right)\right)\left(\frac{2}{n}\right)\right]$$

2 $\frac{1}{3-0}\int_0^3 (x^2 - 2x)\,dx = \frac{1}{3}\left[\frac{1}{3}x^3 - \frac{2}{2}x^2\right]_0^3$

$= \frac{1}{3}\left[\frac{1}{3}(27) - (9) - (0-0)\right]$

$= \frac{1}{3}[9-9] = 0 = f_{AVG} \overset{SET}{=} x^2 - 2x \Rightarrow$

$x \in \{0, 2\} \Rightarrow$

$\boxed{c = 2}$

$(0 \notin (0,3))$