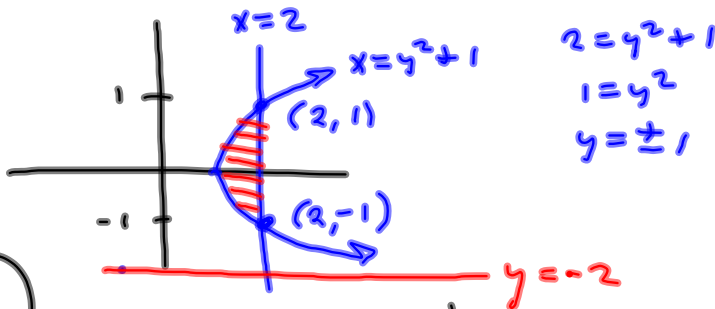


§5.3 #20

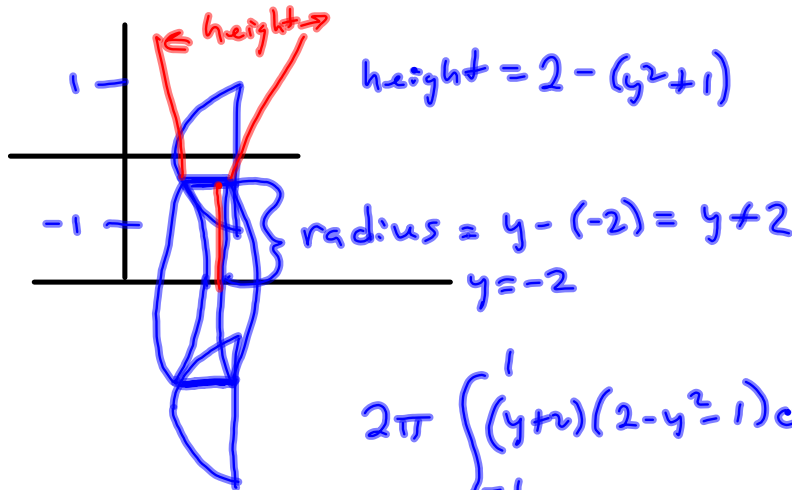
Volume of solid of revolution obtained when  $x=y^2+1$ ,  $x=2$  region is rotated about  $y=-2$



M1

Shell Method

$$2\pi \int_a^b y f(y) dy$$



$$2\pi \int_{-1}^1 (y+2)(2-y^2-1) dy$$

$$= 2\pi \int_{-1}^1 (y+2)(1-y^2) dy$$

$$= 2\pi \int_{-1}^1 (y - y^3 + 2 - 2y^2) dy = 2\pi \left[ \int_{-1}^1 (y - y^3) dy + \int_{-1}^1 (2 - 2y^2) dy \right]$$

(odd fn symmetric interval)

$$= 2\pi \int_{-1}^1 (2 - 2y^2) dy = 4\pi \int_{-1}^1 (1 - y^2) dy$$

$$= 8\pi \int_0^1 (1 - y^2) dy \quad \left\{ \begin{array}{l} \text{Even function} \\ \text{Symmetric Interval} \end{array} \right. \text{Symmetry is useful!}$$

$$= 8\pi \left[ y - \frac{1}{3}y^3 \right]_0^1 = 8\pi \left[ 1 - \frac{1}{3} \right] = \frac{2}{3} \cdot 8\pi = \frac{16\pi}{3}$$

Solve :

$$2 \cos^2 x + \cos x - 1 = 0 \quad \} \text{Homework}$$

$$\textcircled{1} \quad 2 \csc^2 x + \csc x - 1 = 0$$

$$\left. \begin{aligned} \cos^2 x - \sin^2 x &= 0 \\ \tan^2 x - 3 &= 0 \\ \cot^2 x - 3 &= 0 \end{aligned} \right\} \text{Homework}$$

$$2 \sec^2 x + \sec x - 1 = 0$$

Find  $\frac{dy}{dx}$  :

$$x^2 y^3 - 5x \cos y = 2xy + 5 \sin x \quad \} \text{Homework}$$

$$\textcircled{2} \quad x^3 y^2 - 4y \sec y = 3x^2 y - 5 \cos x$$

$$\textcircled{1} \quad 2 \csc^2 x + \csc x - 1 = 0$$

$$2u^2 + u - 1 = 0$$

$$a=2, b=1, c=-1$$

$$b^2 - 4ac = 1^2 - 4(2)(-1) = 9$$

$$\sqrt{a} = 3$$

$$u = \frac{-1 \pm 3}{2(2)} \rightarrow \begin{cases} \frac{2}{4} = \frac{1}{2} \\ -\frac{4}{4} = -1 \end{cases}$$

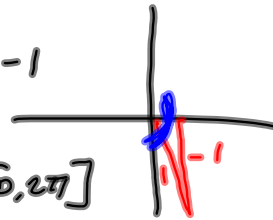
$$u = \csc x = \frac{1}{2}$$

Never!

$$u = \csc x = -1$$

$$u = \frac{3\pi}{2}$$

Answers on  $[0, 2\pi]$



$$\csc x = -1$$

$$\frac{1}{\sin x} = -1$$

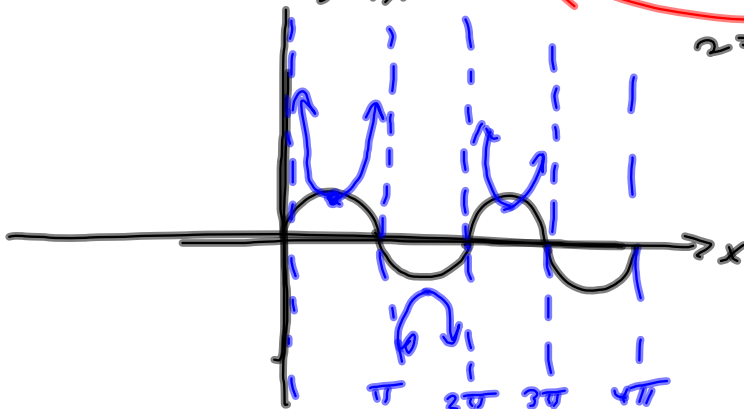
$$1 = -\sin x$$

$$-1 = \sin x$$

$$\csc x = \frac{1}{2}$$

$$\frac{1}{\sin x} = \frac{1}{2}$$

$$2 = \sin x \quad \text{Never!}$$



$\frac{d}{dx}$ 

$$(fg)' = f'g + fg'$$

$$\textcircled{2} \quad x^3 y^2 - 4y \sec y = 3x^2 y - 5 \cos x$$

 $f, g$ 

$f = x^3$

$f' = 3x^2$

$g = y^2$

$g' = 2y y'$

Implicit Differentiation

$$\frac{d}{dx} [x^3 y^2] = 3x^2 y^2 + x^3 (2y y')$$

$$f'g + f g'$$



$$3x^2 y^2 + 2x^3 y y' - 4y' \sec y - 4y (\sec y \tan y) y' = 6xy + 3x^2 y' + 5 \sin x$$

$$2x^3 y y' - 4y' \sec y - 4y (\sec y \tan y) y' - 3x^2 y' = -3x^2 y^2 + 6xy + 5 \sin x$$

$$y' (2x^3 y - 4 \sec y - 4y \sec y \tan y - 3x^2) = -3x^2 y^2 + 6xy + 5 \sin x$$

$$y' = \frac{-3x^2 y^2 + 6xy + 5 \sin x}{2x^3 y - 4 \sec y - 4y \sec y \tan y - 3x^2}$$

Find equation of tangent line to

$$f(x) = \cos x \quad \text{at } x = 60^\circ = \frac{\pi}{3} \text{ radians.}$$

$$f'(x) = -\sin x \quad (60^\circ) \left( \frac{\pi \text{ radians}}{180^\circ} \right)$$

$$f\left(\frac{\pi}{3}\right) = \cos \frac{\pi}{3} \quad \begin{array}{c} \sqrt{3} \\ \swarrow \downarrow \searrow \\ 1 \end{array}$$

$$= \frac{1}{2} \rightsquigarrow \left(\frac{\pi}{3}, \frac{1}{2}\right) = (x_1, y_1)$$

$$f'\left(\frac{\pi}{3}\right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2} = m$$

$$y = m(x - x_1) + y_1$$

$$= -\frac{\sqrt{3}}{2} \left(x - \frac{\pi}{3}\right) + \frac{1}{2}$$

Estimate  $\cos(65^\circ)$  this way.

$$65^\circ = 60^\circ + 5^\circ = \frac{\pi}{3} + 5^\circ = \frac{\pi}{3} + \frac{5\pi}{180}$$

$$\cos(65^\circ)$$

$$\approx y(65^\circ) = -\frac{\sqrt{3}}{2} \left(\frac{\pi}{3} + \frac{\pi}{36} - \frac{\pi}{3}\right) + \frac{1}{2} \quad \begin{array}{c} \Delta x \\ \downarrow \end{array} \quad = \frac{\pi}{3} + \frac{\pi}{36} \text{ radians}$$

$$= -\frac{\sqrt{3}}{2} \left(\frac{\pi}{36}\right) + \frac{1}{2}$$

$$= \frac{-\sqrt{3}\pi + 36}{2}$$