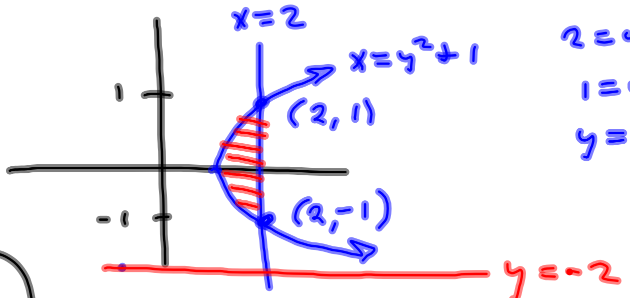
§5.3 #20

volume of solid of revolution obtained when $x=y^2+1$, $x=2$ region is rotated about $y=-2$

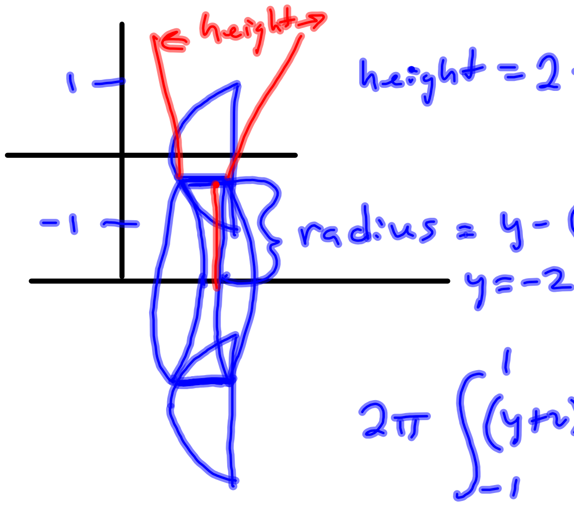


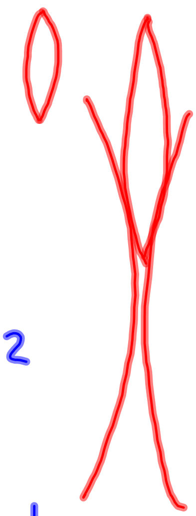
$2=y^2+1$

$1=y^2$

$y=\pm 1$

M1 Shell Method $\quad 2\pi\int_a^b y\, f(y)\, dy$





height $= 2-(y^2+1)$

radius $= y-(-2) = y+2$

$$2\pi\int_{-1}^{1}(y+2)(2-y^2-1)\,dy$$

$$= 2\pi\int_{-1}^{1}(y+2)(1-y^2)\,dy$$

$$= 2\pi\int_{-1}^{1}(y-y^3+2-2y^2)\,dy = 2\pi\left[\int_{-1}^{1}(y-y^3)\,dy + \int_{-1}^{1}(2-2y^2)\,dy\right]$$

0 (odd fn symmetric interval)

$$= 2\pi\int_{-1}^{1}(2-2y^2)\,dy = 4\pi\int_{-1}^{1}(1-y^2)\,dy$$

$$= 8\pi\int_{0}^{1}(1-y^2)\,dy \quad \text{(Even function Symmetric Interval)}$$

Symmetry is useful!

$$= 8\pi\left[y-\tfrac{1}{3}y^3\right]_0^1 = 8\pi\left[1-\tfrac{1}{3}\right] = \tfrac{2}{3}\cdot 8\pi = \tfrac{16\pi}{3}$$

Nov 20-12:13 PM

Solve :

$2\cos^2 x + \cos x - 1 = 0$ } Homework

① $2\csc^2 x + \csc x - 1 = 0$

$\cos^2 x - \sin^2 x = 0$

$\tan^2 x - 3 = 0$ } Homework

$\cot^2 x - 3 = 0$

$2\sec^2 x + \sec x - 1 = 0$

find $\frac{dy}{dx}$:

$x^2 y^3 - 5x \cos y = 2xy + 5\sin x$ } Homework

② $x^3 y^2 - 4y \sec y = 3x^2 y - 5\cos x$

1. $2\csc^2 x + \csc x - 1 = 0$

$2u^2 + u - 1 = 0$

$a = 2,\ b = 1,\ c = -1$

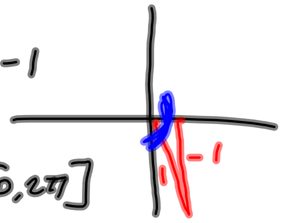
$b^2 - 4ac = 1^2 - 4(2)(-1)$
$= 9$

$\sqrt{9} = 3$

$u = \dfrac{-1 \pm 3}{2(2)}$ → $\dfrac{2}{4} = \dfrac{1}{2}$; $\dfrac{-4}{4} = -1$

$u = \csc x = \frac{1}{2}$
Nevah!

$u = \csc x = -1$
$u = \frac{3\pi}{2}$
Answers on $[0, 2\pi]$



$\csc x = -1$

$\frac{1}{\sin x} = -1$

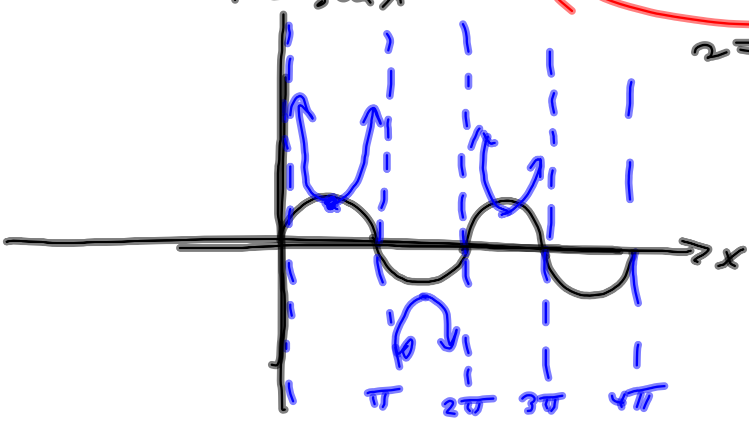
$1 = -\sin x$

$-1 = \sin x$

$\csc x = \frac{1}{2}$

$\frac{1}{\sin x} = \frac{1}{2}$

$2 = \sin x$ Nevah!



$$\frac{d}{dx}$$

$$(fg)' = f'g + fg'$$

(2) $x^3y^2 - 4y\sec y = 3x^2y - 5\cos x$

fg

Implicit Differentiation

$f = x^3 \qquad f' = 3x^2$

$g = y^2 \qquad g' = 2yy'$

$$\frac{d}{dx}\left[x^3y^2\right] = \underset{f'g}{3x^2y^2} + \underset{fg'}{x^3(2yy')}$$

$$3x^2y^2 + 2x^3yy' - 4y'\sec y - 4y(\sec y\tan y)y' = 6xy + 3x^2y' + 5\sin x$$

$$2x^3yy' - 4y'\sec y - 4y(\sec y\tan y)y' - 3x^2y' = -3x^2y^2 + 6xy + 5\sin x$$

$$y'(2x^3y - 4\sec y - 4y\sec y\tan y - 3x^2) = -3x^2y^2 + 6xy + 5\sin x$$

$$\boxed{y' = \frac{-3x^2y^2 + 6xy + 5\sin x}{2x^3y - 4\sec y - 4y\sec y\tan y - 3x^2}}$$

Find equation of tangent line to

$f(x) = \cos x$ @ $x = 60^\circ = \frac{\pi}{3}$ radians.

$f'(x) = -\sin x$ $\qquad (60^\circ)\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3}$

$= \frac{1}{2} \rightsquigarrow \left(\frac{\pi}{3}, \frac{1}{2}\right) = (x_1, y_1)$

$f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2} = m$

$y = m(x - x_1) + y_1$

$= -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) + \frac{1}{2}$

Estimate $\cos(65^\circ)$ this way.

$65^\circ = 60^\circ + 5^\circ = \frac{\pi}{3} + 5^\circ = \frac{\pi}{3} + \frac{5\pi}{180}$

$= \frac{\pi}{3} + \frac{\pi}{36}$ radians

$\cos(65^\circ)$

$\approx y(65^\circ) = -\frac{\sqrt{3}}{2}\left(\frac{\pi}{3} + \underbrace{\frac{\pi}{36}}_{\Delta x} - \frac{\pi}{3}\right) + \frac{1}{2}$

$= -\frac{\sqrt{3}}{2}\left(\frac{\pi}{36}\right) + \frac{1}{2}$

$= \frac{-\sqrt{3}\pi + 36}{2}$