§ Appendix E

#s 47, 50

$P(n)$ = statement to be proved $\forall n \in \mathbb{N}$.

Induction: ① Let $S = \{n \mid P(n) \text{ holds}\}$

② Show $1 \in S$ (i.e., $P(1)$ holds)

③ Assume $n \in S$. Use this to show $n+1 \in S$.

Claim: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \forall n \in \mathbb{N}$: $P(n)$

<u>Proof</u>: Let $S = \{n \in \mathbb{N} \mid \sum_{k=1}^{n} k = \frac{n(n+1)}{2}\}$

Since $\sum_{k=1}^{1} k = 1$, $\frac{1(1+1)}{2} = 1$ } So, $1 \in S$

Goal: Show $\sum_{k=1}^{n+1} k = \frac{(n+1)(n+1+1)}{2}$

Assume $n \in S$. Then $P(n)$ holds, i.e.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = 1+2+3+4+\dots+n$$

Consider

Induction step

$$\sum_{k=1}^{n+1} k = \underbrace{1+2+3+\dots+n}_{\sum_{k=1}^{n} k = \frac{n(n+1)}{2}} + (n+1) = \frac{n(n+1)}{2} + n+1$$

$$= \frac{n^2+n}{2} + \frac{2}{2}(n+1) = \frac{n^2+n}{2} + \frac{2n+2}{2} = \frac{n^2+3n+2}{2}$$

$$= \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2} \Longrightarrow n+1 \in S$$

$\Longrightarrow S = \mathbb{N}$ ■

#47 Prove $\sum_{k=1}^{n} ar^{k-1} = a\left(\frac{r^n - 1}{r-1}\right)$

Induction step: $\sum_{k=1}^{n+1} ar^{k-1} = a\left(\frac{r^{n+1} - 1}{r-1}\right)$

#50 Not induction

$\sum_{k=1}^{m}\left(\sum_{j=1}^{n}(k+j)\right) =$

HINT: Addition is commutative

$(1+1)+(1+2)+(1+3)$

$= \underline{1+1+1} + \underline{1+2+3}$

$(1+1)+(1+2)+(1+3)+\cdots+(1+n)$

$+ (2+1)+(2+2)+(2+3)+\cdots+(2+n)$

$+ \cdots$

$+ (m+1)+(m+2)+(m+3)+\cdots+(m+n)$

$$\lim_{x\to 2}\frac{2x^2+x-10}{3x^2-13x+14}=\lim_{x\to 2}\frac{\cancel{(x-2)}(2x+5)}{\cancel{(x-2)}(3x-7)}=\frac{(2(2)+5)}{(3(2)-7)}=\frac{9}{-1}=-9$$

Common Error: Plug in $x=2$, get $\frac{0}{0}$ & throw up your hands.

Another approach, instead of factoring:

$$\begin{array}{r|rrr} 2 & 3 & -13 & 14 \\ & & 6 & -14 \\ \hline & 3 & -7 & \end{array}$$

$\frac{0}{0}$ says $x=2$ is zero of numerator.

$\therefore$ $x-2$ is a factor

$\therefore$ Dividing by $x-2$ reveals the other factor!

$\frac{2x^2}{x}=2x$

$\frac{5x}{x}=5$

$$\begin{array}{r|l} & 2x+5 \\ x-2 & 2x^2+x-10 \\ & -(2x^2-4x) \\ \hline & 5x-10 \\ & -(5x-10) \\ \hline & 0 \end{array}$$

This says $2x^2+x-10$

$$=\underbrace{(x-2)}_{\text{Divisor}}\underbrace{(2x+5)}_{\text{Quotient}}$$

Synthetic: $x=2$ is zero $\Rightarrow$ $x-2$ is factor

$$\begin{array}{r|rrr} 2 & 2 & 1 & -10 \\ & \downarrow & 4 & 10 \\ \hline & 2 & 5 & 0 \\ & x' & c & r \end{array}$$

$\Rightarrow 2x^2+x-10$

$=(x-2)(2x+5)$

Review Find $f'(x)$ by the limit definition

for $f(x) = 2x^2 - 5x$

Bonus $f(x) = x^3 - 4x$

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h}, \text{etc.}$$

Bonus $$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - 4(x+h) - (x^3 - 4x)}{h}$$

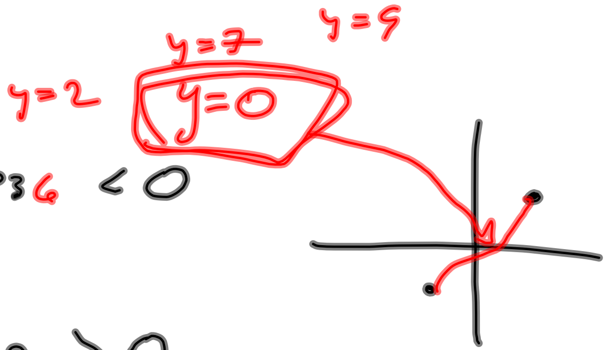
Prove that

$3\sin x - x$ has a zero in $(-\frac{\pi}{4}, \frac{\pi}{2})$

$$f(-\tfrac{\pi}{4}) = 3\sin(-\tfrac{\pi}{4}) - (-\tfrac{\pi}{4})$$

$$= 3(-\tfrac{\sqrt{2}}{2}) + \tfrac{\pi}{4}$$

$$= -\tfrac{3\sqrt{2}}{2} + \tfrac{\pi}{4} \approx -1.336 < 0$$



$$f(\tfrac{\pi}{2}) = 3\sin(\pi/2) - \tfrac{\pi}{2}$$

$$= 3 - \tfrac{\pi}{2} \approx 1.429 > 0$$

$3\sin x - x$ is cnt^s; $\therefore$ it crosses the x-axis between $-\frac{\pi}{4}$ & $\frac{\pi}{2}$, by Intermediate Value Theorem. IVT

Derivatives, Antiderivatives

$$\frac{d}{dx}\sin(\cos(\tan(x^2-5x))) =$$

$$\cos(\cos(\tan(x^2-5x)))\cdot(-\sin(\tan(x^2-5x)))(\sec^2(x^2-5x))(2x-5)$$

$$\frac{d}{dx}\left(\sin\left(\frac{x^2-3x}{\cos x}\right)\right)(2x+1)$$

$$= \cos\left(\frac{x^2-3x}{\cos x}\right)\left(\frac{(2x-3)(\cos x)-(x^2-3x)(-\cos x)}{\cos^2 x}\right)(2x+1)$$

$$+\left(\sin\left(\frac{x^2-3x}{\cos x}\right)\right)(2)$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$