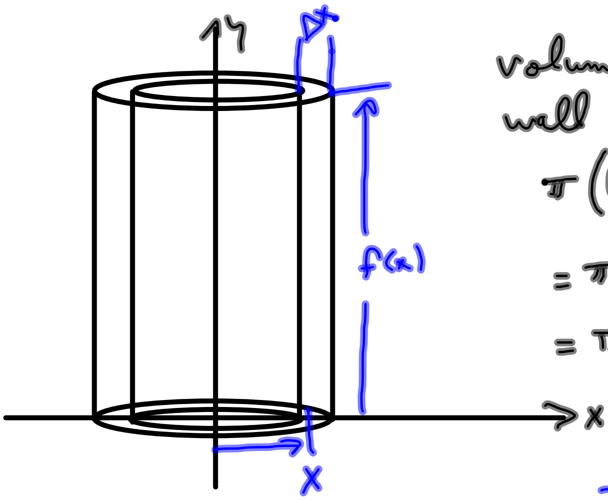


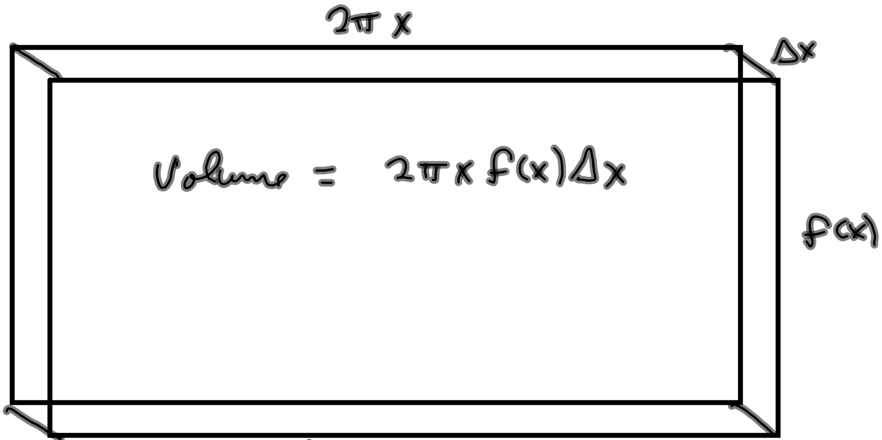
volume of the cylinder wall is

$$\pi\left((x+\Delta x)^2\right) f(x) - \pi\left(x^2\right) f(x)$$

$$= \pi f(x)\left(x^2 + 2\Delta x + (\Delta x)^2 - x^2\right)$$

$$= \pi f(x)\left(2\Delta x + (\Delta x)^2\right)$$

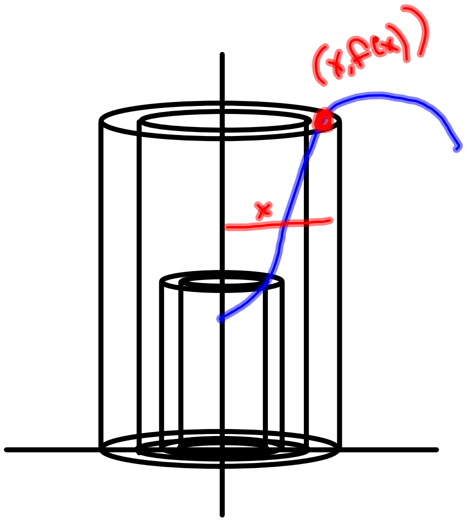
Slice it vertically. Lay it out flat.

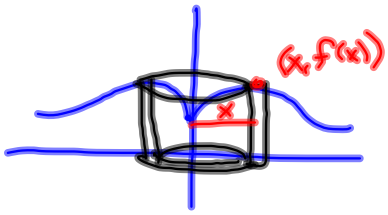


$$\text{Volume} = \lim_{n\to\infty} \sum_{k=1}^{n} 2\pi x_k f(x_k)\Delta x$$

$$= 2\pi \int_a^b x f(x_k)\Delta x$$

For revolving $y=f(x)$ about the y-axis.

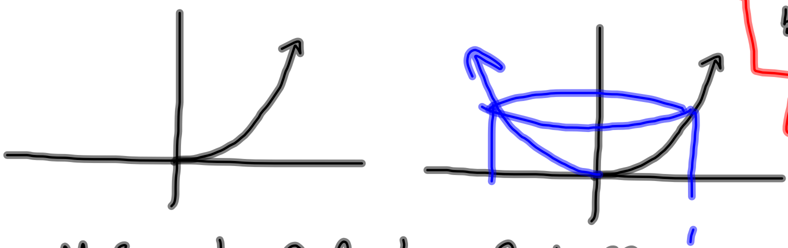




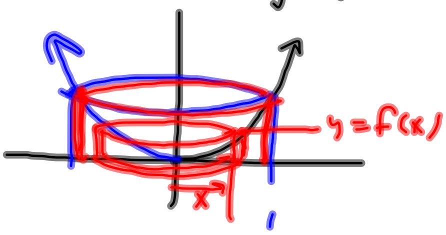
$\boxed{E}$ Revolve
$y=x^2$ about the y-axis.
$x=0$, to $x=1$, $y=0$

$$= 2\pi\int_a^b x f(x)\,dx$$

Region bounded by $y=x^2$, $y=0$, $x=0$, $x=1$



Volume by Cylindrical Shells.



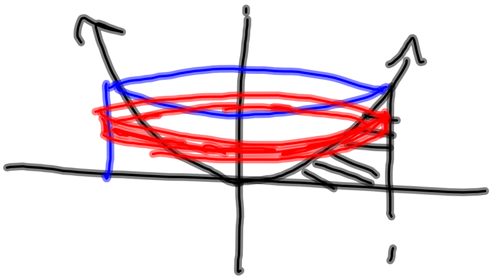
$$V = 2\pi\int_a^b x f(x)\,dx$$

$$= 2\pi\int_0^1 x\cdot x^2\,dx$$

$$= 2\pi\int_0^1 x^3\,dx$$

$$= 2\pi\left[\frac{1}{4}x^4\right]_0^1 = \frac{\pi}{2}$$

Same thing by
Disk Method will
require Washer Method!



$r=$

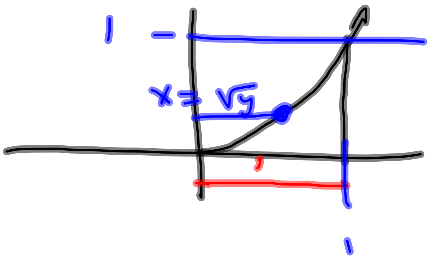
Discs lay flat.
This is an $x=g(y)$ & dy
situation

$y=x^2$
$x=\pm\sqrt{y}$. Use positive $x=\sqrt{y}$

volume of washer = Outer Disk - Inner Disk

$(\sqrt{y})^2=y$

$\sqrt{y^2}=|y|$



$$\pi\int_0^1 \left(1^2-(\sqrt{y})^2\right)dy$$

$$\pi\int_0^1 (1-y)\,dy$$

$$=\pi\left[y-\tfrac{1}{2}y^2\right]_0^1 dy$$

$$=\pi\left[1-\tfrac{1}{2}\right]=\frac{\pi}{2}$$

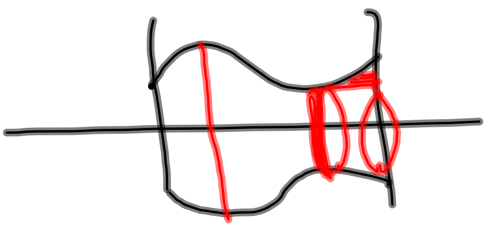
§5.3 #s 3, 6, 8, 9, 12, 15, 19, 20

When does washer method suck?

Rotate f(x) about y-axis, from 0 to 7

f(x) isn't 1to1 so $x=g(y)$ is a pain. A lot of different washers - Pain.

Cylindrical Shells can suck, when it's $y=f(x)$ rotated about the x-axis



Requires $x=g(y)$ for a function that's not 1-to-1.

11, 13, 31

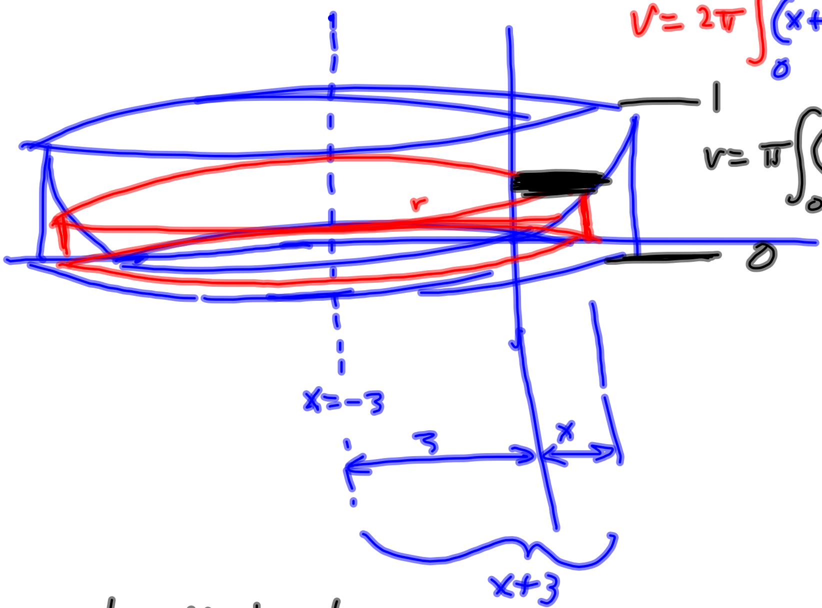
Find Volume when

Rotate $y = x^2$, bdd by $y=0$, $x=1$, $x=0$

... about $x=-3$ $\qquad y = x^2 \Rightarrow x = \sqrt{y}$

... about $y=-3$

$V = 2\pi \int_0^1 (x+3)\, x^2\, dx$



$v = \pi \int_0^1 \left(4^2 - (\sqrt{y})^2\right) dy$

Nope.

$\sqrt{y} - (-3)$ is the radius of the inner disc.

Washer Method:

$$\pi \int_0^1 4^2\, dy - \pi \int_0^1 (\sqrt{y}+3)^2\, dy$$