§5.1 #12 another way

Last time graphed x as a func. of y.

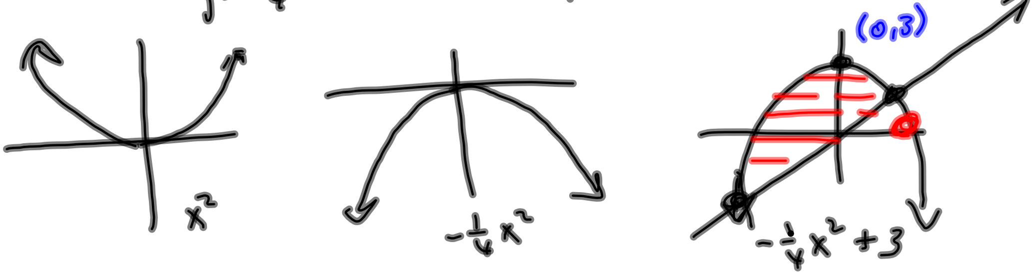
Another way is to swap variables.

$4x+y^2=12$ $\qquad x=y$

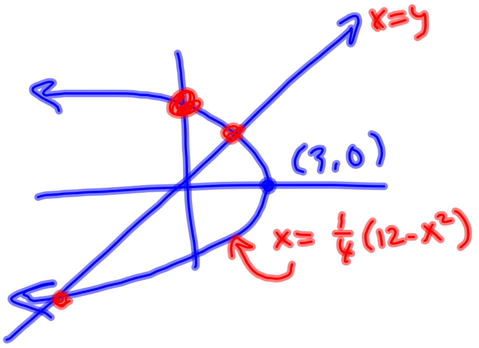
$4y+x^2=12$ $\qquad y=x$

$4y=12-x^2$

$y=\frac{1}{4}(12-x^2)=3-\frac{1}{4}x^2$

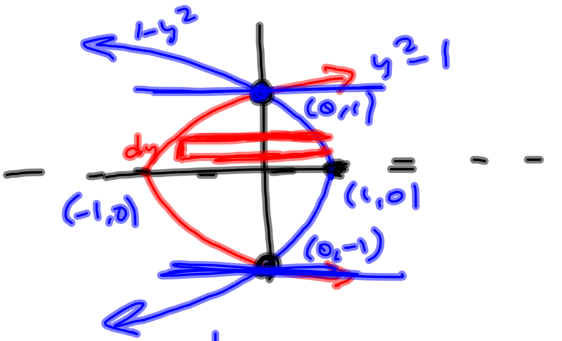


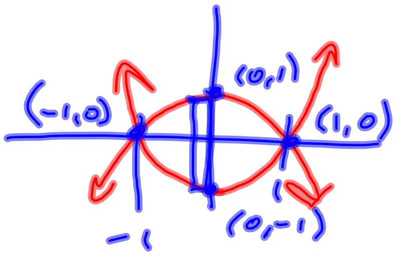
Now, the pic for $x=\frac{1}{4}(12-y^2)$ & $\boxed{x=y}$ might be easier.



#4 $x = 1-y^2$, $x = y^2 - 1$

$y = 1-x^2$, $y = x^2 - 1$





$$\text{Area} = \int_{-1}^{1} \left((1-y^2) - (y^2-1) \right) dy$$

$$\int_{-1}^{1} (1-x^2 - (x^2-1))\,dx$$

Symmetry:

$$\text{Area} = 2\int_0^1 (1-y^2 - (y^2-1))\,dy$$

Pg 329 Newton and Liebniz
Good bit of history.

§5.2 Volumes

Think of it as finding area, sort of like before, multiply by thickness (dx) and you have a volume.

$$V \approx \sum_{k=1}^{n} A(x_k)\Delta x \xrightarrow{n\to\infty} \int_a^b A(x)\,dx$$

These used to be
$f(x) = \text{height}(x)$ function
Now we have $f(x) = \text{Area}(x)$

Nov 6-12:26 PM

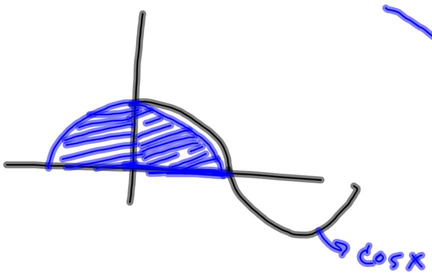
Volume of cylinder

$\underbrace{\pi r^2}_{\text{Area of base}} \underbrace{h}_{\text{height}}$

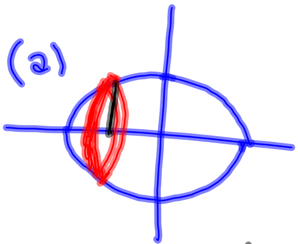
§5.2 # 32 $y=0$, $y=\cos^2 x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Rotate about (a) x-axis

(b) y-axis



(a)

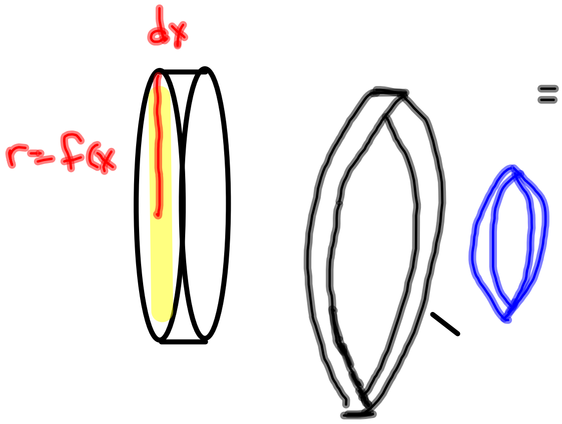


$V = \pi r^2 h$

$= \pi f(x)^2 dx$

volume of the disk is

$$V = \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^2 x)^2 dx$$



$$= 2\pi \int_0^{\frac{\pi}{2}} \cos^4 x \, dx$$ Done

How to break it down?

$$= 2\pi \int_0$$

$u = \cos x$

$du = -\sin x \, dx$ is hard to generate.

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$(\cos^2\theta)^2 = \left(\frac{1+\cos(2\theta)}{2}\right)^2 = \frac{1}{4}\left(\cos^2(2\theta) + 2\cos(2\theta) + 1\right)$$

$$= \frac{1}{4}\left[\frac{1+\cos(4\theta)}{2} + 2\cos(2\theta) + 1\right]$$

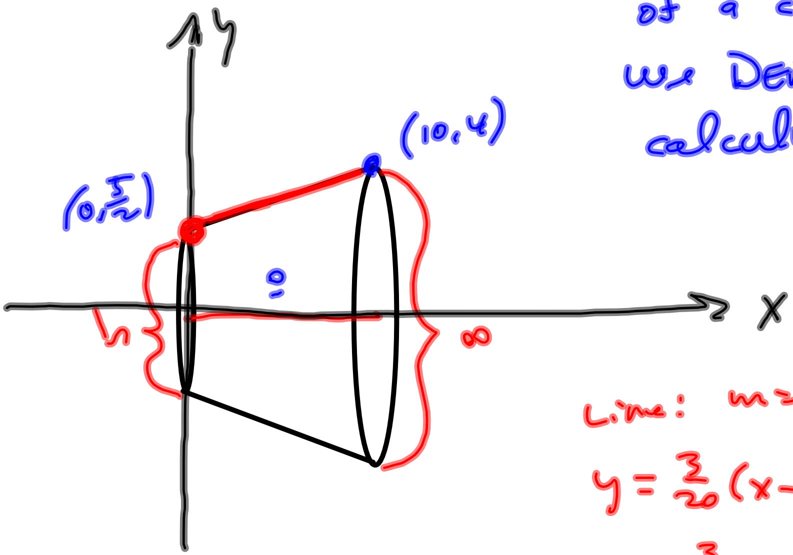
etc.

$u = 4\theta \Rightarrow du = 4d\theta$

$$\int \cos(4\theta)\,d\theta = \frac{1}{4}\int \cos(4\theta)\cdot 4\,d\theta$$

Find the volume

Volume of a frustum of a cone has a formula. We DERIVE it with calculus.



Line: $m = \frac{4 - 5/2}{10 - 0} = \frac{\frac{3}{2}}{10} = \frac{3}{20}$

$y = \frac{3}{20}(x - 0) + \frac{5}{2}$

$y = \frac{3}{20}x + \frac{5}{2}$

$\therefore$ Volume $= \pi \int_0^{10} \left(\frac{3}{20}x + \frac{5}{2}\right)^2 dx$

§5.2 #'s 31, [illegible] 11, 13

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int \sec x\,dx = \int \frac{\sec x \cdot (\sec x + \tan x)}{\sec x + \tan x}\,dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$$

$u = \sec x + \tan x$

$du = (\sec^2 x + \sec x \tan x)\,dx$

$$= \int \frac{du}{u}$$