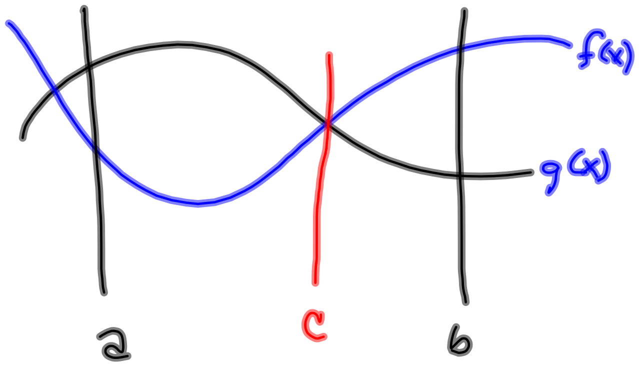
8a $\dfrac{x^2-3x}{\csc^3 x} = \dfrac{d}{dx}\displaystyle\int_{\frac{\pi}{6}}^{x} \frac{t^2-3t}{\csc^3 t}\,dt$

8b $\dfrac{(x^3)^2-3x^3}{\csc^3(x^3)} \cdot 3x^2$

§5.1 Areas between / bounded by curves



Area bdd by $f(x)$ & $g(x)$ from $x=a$ to $x=b$ (over $[a,b]$)

$$\int_a^b |f(x)-g(x)|\,dx$$

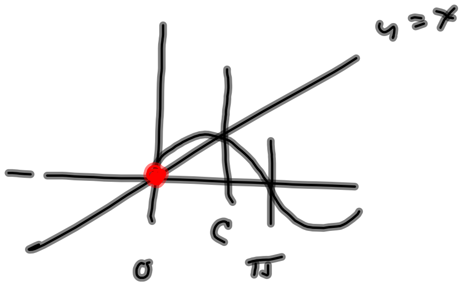
$$= \int_a^c (g(x)-f(x))\,dx + \int_c^b (f(x)-g(x))\,dx$$

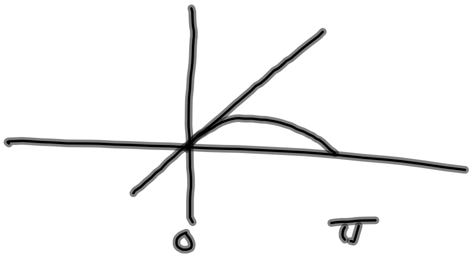
Area between $f(x)=x$ & $g(x)=\sin x$ on $[\frac{\pi}{2}, \pi]$

Hand sketch skills matter.

Technology is there, if needed on homework.

Nov 5-12:11 PM





which picture?
If 1st picture, where is c?

They share (0,0).

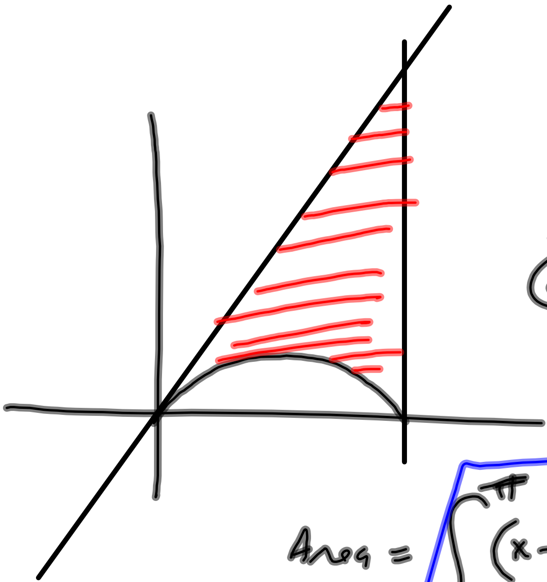
$y=x$ $y=\sin x$

$\boxed{y'=1}$ $y'=\cos x$

$\boxed{y'(0)=1}$ &

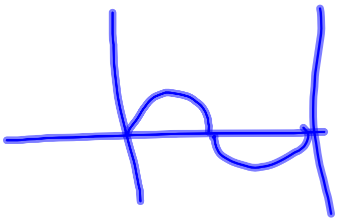
Slopes equal @ start.

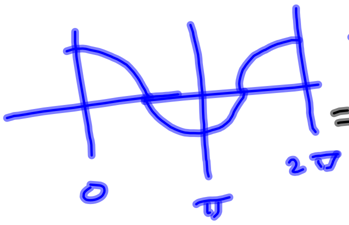
$y''(x) = -\sin x$ is negative on $(0,\pi)$



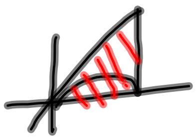
Slope of $y=\sin x$ is decreasing.

$$\text{Area} = \boxed{\int_0^{\pi} (x-\sin x)\,dx}$$





$$= \int_0^{\pi} x\,dx - \int_0^{\pi} \sin x\,dx$$



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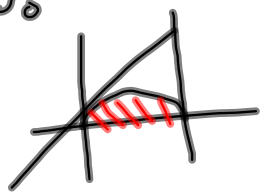


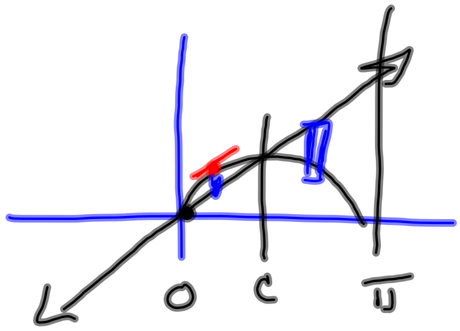
$$= \left[\tfrac{1}{2}x^2 + \cos x\right]_0^{\pi} = \tfrac{1}{2}\pi^2 + (-1) - \left(\tfrac{1}{2}(0)^2 + \cos(0)\right)$$

$$= \boxed{\tfrac{1}{2}\pi^2 - 2 = \text{Area}}$$

Same question, but $f(x)=\frac{1}{2}x$ & $g(x)=\sin x$

$f'(x)=\frac{1}{2}$ $g'(x)=\cos x$

$f'(0)=\frac{1}{2}$ $g'(0)=1$



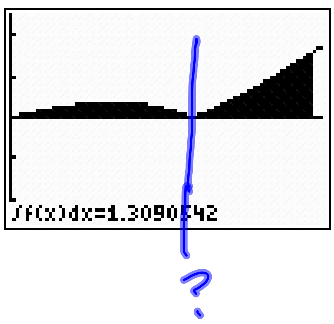
Find c: $\frac{1}{2}x = \sin x$ No paper-and-pencil (analytic) solution.

Other methods!

$\frac{1}{2}x - \sin x = 0$ Graphing Calculator.

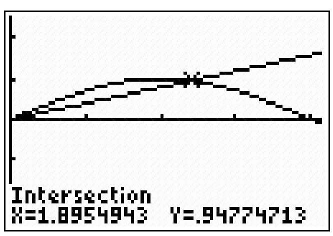
Newton's Method.

$$\text{Area} = \int_0^c \left(\sin x - \tfrac{1}{2}x\right)dx + \int_c^{\pi}\left(\tfrac{1}{2}x - \sin x\right)dx$$



Graphing Crutch:

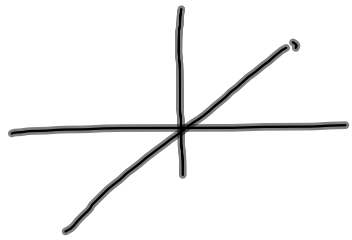
$$\int_0^{\pi} \left|\tfrac{1}{2}x - \sin x\right| dx \approx 1.3090542$$



$$\int_0^{1.8954943}\left(\sin x - \tfrac{1}{2}x\right)dx + \int_{1.8954943}^{\pi}\left(\tfrac{1}{2}x - \sin x\right)dx$$

#12

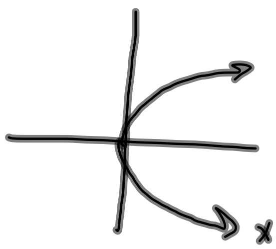
Area between $4x+y^2=12$, $x=y$



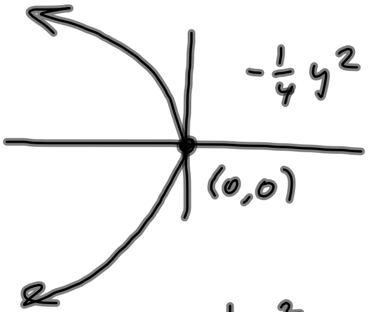
$4x = 12-y^2$

$x = \frac{12-y^2}{4}$

$x = 3 - \frac{1}{4}y^2$

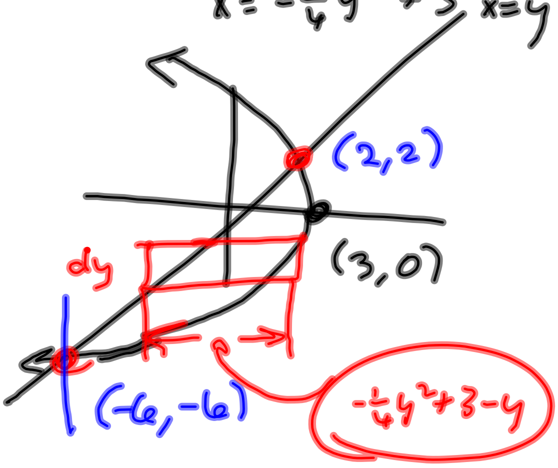


$x=y^2$



$-\frac{1}{4}y^2 = x$

$x = -\frac{1}{4}y^2 + 3$ $x=y$



$x=x$

$-\frac{1}{4}y^2+3 = y$

$-\frac{1}{4}y^2 - y + 3 = 0$

$y^2 + 4y - 12 = 0$

$(y+6)(y-2) = 0$

$y = -6, 2$

$$\text{Area} = \int_{-6}^{2} \left(\left(-\frac{1}{4}y^2+3\right) - y\right) dy$$

#12

Area between $4x+y^2=12$, $x=y$

$y^2 = 12-4x$ $\qquad$ $y=x$

$y = \pm\sqrt{12-4x}$

$= \pm 2\sqrt{3-x}$

$2\sqrt{3-x} =$

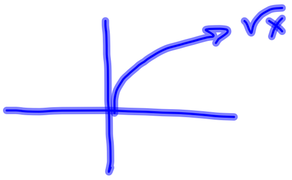
$2\sqrt{-(x-3)}$

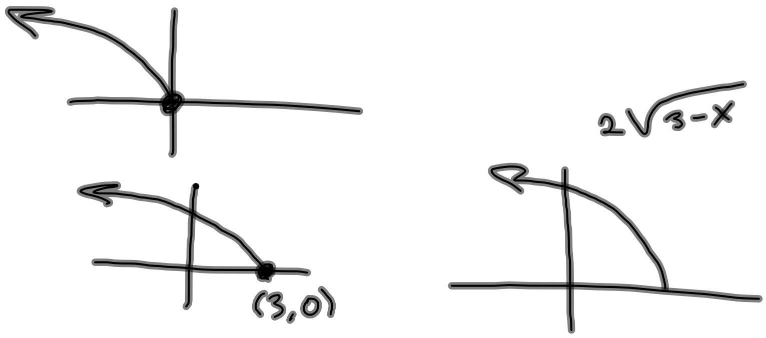
① $\sqrt{x}$

② $\sqrt{-x}$

③ $\sqrt{-(x-3)}$

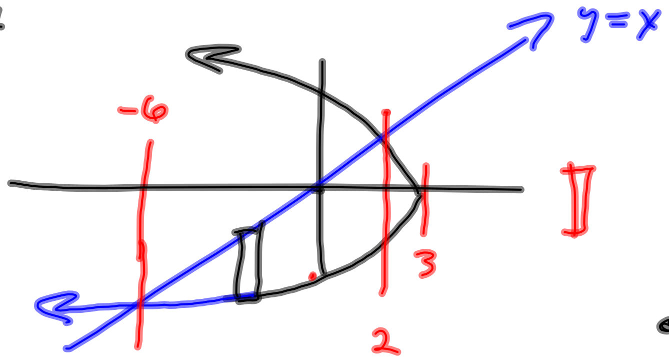
④ $2\sqrt{-(x-3)}$





$y = \pm 2\sqrt{3-x}$

§5.1 #s
5, 9, 11, 13, 15, 17,
19, 23, 31, 35, 37



$\int$ upper - lower = ?

$= \int_{-6}^{2} \left(x - (-2\sqrt{3-x})\right) dx + \int_{2}^{3} \left(2\sqrt{3-x} - (-2\sqrt{3-x})\right) dx$

$= \int_{0}^{2} + \quad 4\int_{2}^{3} \sqrt{3-x}\ dx$