§4.4 # 39

$$\int_2^5 |x-3|\,dx$$

$$|*| = \begin{cases} * & \text{if } * \geq 0 \\ -* & \text{if } * < 0 \end{cases}$$

$x-3 \geq 0$

$x \geq 3 \implies$

$$|x-3| = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$$

$$\int_2^3 -(x-3)\,dx + \int_3^5 (x-3)\,dx$$

$$= -\left[\tfrac{1}{2}x^2 - 3x\right]_2^3 + \left[\tfrac{1}{2}x^2 - 3x\right]_3^5$$

$$= -\left[\tfrac{1}{2}(3)^2 - 3(3) - \left(\tfrac{1}{2}(2)^2 - 3(2)\right)\right] +$$

$$\tfrac{1}{2}(5)^2 - 3(5) - \left(\tfrac{1}{2}(3)^2 - 3(3)\right)$$

$$= -\left(\tfrac{9}{2} - 9 - (2-6)\right) + \left(\tfrac{25}{2} - 15 - \left(\tfrac{9}{2} - 9\right)\right)$$

$$= -\tfrac{9}{2} + 9 - 4 \qquad + \tfrac{25}{2} - 15 - \tfrac{9}{2} + 9$$

$$= -\tfrac{9}{2} - \tfrac{9}{2} + \tfrac{25}{2} + 5 - 6$$

$$= -\tfrac{18}{2} + \tfrac{25}{2} - \tfrac{2}{2} = \tfrac{5}{2}$$

§4.4 #48

$I(t) = Q'(t)$

$$\int_a^b I(t)\,dt = Q(b) - Q(a) = \int_a^b Q'(t)\,dt$$

is Net Change in Charge.

#49 $\int_0^{120} r(t)\,dt$ = Amt of oil spilled in 2 hrs.

Net amount

Assume $r(t) \geq 0$

So it's total amt of oil spilled.

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§ 4.5 # 15

$$\int \frac{bx^2+a}{\sqrt{bx^3+3ax}}\,dx$$

$u = bx^3+3ax$

$du = (3bx^2+3a)\,dx$

$= 3(bx^2+a)\,dx$

$dx = \dfrac{du}{3(bx^2+a)}$

$= \frac{1}{3}\int (bx^3+3ax)^{-\frac{1}{2}}\, 3(bx^2+a)\,dx$

$= \int (bx^3+3ax)^{-\frac{1}{2}} \cancel{(bx^2+ax)} \dfrac{du}{3\cancel{(bx^2+a)}}$

$= \int u^{-\frac{1}{2}} \cdot \dfrac{du}{3} = \frac{1}{3}\int u^{-\frac{1}{2}}\,du$

$= \frac{1}{3}\int u^{-\frac{1}{2}}\,du = \frac{1}{3}\,\dfrac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3}u^{\frac{1}{2}} + C$

$= \frac{1}{3}\,\dfrac{(bx^3+3ax)^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3}(bx^3+3ax)^{\frac{1}{2}} + C$

#21 §4.5

$$-\frac{d}{dx}[\csc x] = +\csc x \cot x$$

$$\int \frac{\cos x}{\sin^2 x}\,dx = \int \underline{\cot x \csc x}\,dx$$

$$= -\csc x + C$$

$u = \sin x$
$du = \cos x\,dx \Longrightarrow$

$$\int \frac{du}{u^2} = \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1} + C = -(\sin x)^{-1} + C$$

$$= -\csc x + C$$

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Error in §4.5 sol'ns #42

$$\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\,dx = \int_0^{\frac{\pi}{2}} \sin(\sin x)\cos x\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$= \int_0^{\frac{\pi}{2}} \sin(u)\,du$ is wrong.

These are "x="

$\int_0^1 \sin(u)\,du$ is correct.

$x = 0 \Rightarrow u = \sin(0) = 0$

$x = \frac{\pi}{2} \Rightarrow u = \sin(\frac{\pi}{2}) = 1$

$= \left[-\cos(u)\right]_0^1$

$= -\cos(1) - (-\cos(0))$

$= -\cos(1) + 1$

$\approx .4596976941$

Make sure you're in radians mode!

$\frac{\pi}{4} < 1$ $\frac{\pi}{3} > 1$

$\frac{\pi}{4} < 1 < \frac{\pi}{3}$

1 radian is between $60°$ & $45°$

$1°$ is teensy

Trig Idents to know

$$\boxed{\begin{aligned} &\cos^2 x + \sin^2 x = 1 \\ &\cot^2 x + 1 = \csc^2 x \\ &\tan^2 x + 1 = \sec^2 x \end{aligned}}$$

$$\boxed{\cos^2 x = \frac{1 + \cos(2x)}{2}}$$

$$\sin^2 x = 1 - \cos^2 x = 1 - \left(\frac{1+\cos(2x)}{2}\right) = \frac{1-\cos(2x)}{2}$$

$$\frac{1}{n}\left[\left(\frac{1}{n}\right)^9+\left(\frac{2}{n}\right)^9+\cdots+\left(\frac{n}{n}\right)^9\right] \approx \int_0^1 x^9\,dx$$

$$= \frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^9 = \frac{1}{n}\cdot\frac{1}{n^9}\sum_{k=1}^{n} k^9 = \frac{1}{n^{10}}\left[\frac{n^{10}+\sim\sim}{10}\right]$$

$$= \frac{n^{10}}{10n^{10}} + \underbrace{\frac{\sim\sim}{10n^{10}}}_{\to\, 0} \xrightarrow{n\to\infty} \frac{1}{10}$$

"$\sim\sim$" means terms of lower degree.

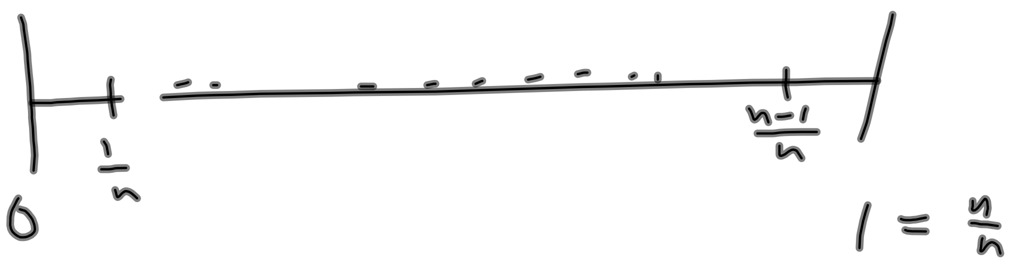
Hard to "see" it as an integral.
But if it IS, then $\frac{1}{n}$ is acting like Δx

$$\frac{b-a}{n} = \frac{1}{n}$$

$$x_1 = \frac{1}{n},\ x_2 = \frac{2}{n},\ \ldots,\ x_n = \frac{n}{n}$$

$$x_k = a + k\Delta x = \frac{k}{n}$$

$$a = 0,\ \Delta x = \frac{1}{n}$$



$$y = \int_{\sqrt{x}}^{x} \frac{\cos\theta}{\theta}\, d\theta = \int_0^x \frac{\cos\theta}{\theta}\, d\theta + \int_{\sqrt{x}}^{0} \frac{\cos\theta}{\theta}\, d\theta$$

$$\frac{dy}{dx} =$$

Nah use 1 instead of 0,
b/c issues @ $\theta = 0$, $\sqrt{x} \geq 0$, etc.

$$= \int_1^x \frac{\cos\theta}{\theta}\, d\theta - \int_1^{\sqrt{x}} \frac{\cos\theta}{\theta}\, d\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{x} - \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

Chain Rule Part.

$$\int_1^3 \sqrt{x^2+3}\,dx$$

$$\frac{d}{dx}(x^2+3)^{\frac{1}{2}} = \frac{1}{2}(x^2+3)^{-\frac{1}{2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+3}} \geq 0 \text{ always.}$$

so $\sqrt{1^2+3} = \sqrt{4} = 2 \leq \sqrt{x^2+3}$ on $[1,3]$

$\sqrt{3^2+3} = \sqrt{12} = 2\sqrt{3} \geq \sqrt{x^2+3}$ on $[1,3]$

$$\therefore\ 2[3-1] \leq \int_1^3 \sqrt{x^2+3}\,dx \leq 2\sqrt{3}[3-1]$$

$$4 \leq \int_1^3 \sqrt{x^2+3}\,dx \leq 4\sqrt{3}$$

$M(b-a)$ Big Rect.

$m(b-a)$ = small rect.