§4.5 #51

$$\int_0^1 \frac{dx}{(\sqrt{x}+1)^4}$$

$u = \sqrt{x} + 1$

NOT $(\sqrt{x}+1)^4$, silly.

No!

$u = \sqrt{x} + 1 = x^{\frac{1}{2}} + 1$

$du = \frac{1}{2}x^{-\frac{1}{2}}dx = du$

$dx = 2x^{\frac{1}{2}}du$

$x=0 \Rightarrow u = (0+1)^4 = 1$

$x=1 \Rightarrow u = (\sqrt{1}+1)^4 = 16$

Good job, David & Spencer.

No $x=1 \Rightarrow u = \sqrt{1}+1 = 2$

$$\Rightarrow \int_1^2 \frac{2x^{\frac{1}{2}}du}{u^4} = 2\int_1^2 \frac{u-1}{u^4}du = 2\int_1^2 (u^{-3}-u^{-4})du$$

$\left(u = x^{\frac{1}{2}}+1 \Rightarrow u-1 = x^{\frac{1}{2}}\right)$

$$= 2\left[\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3}\right]_1^2 = 2\left[-\frac{1}{2u^2} + \frac{1}{3u^3}\right]_1^2$$

$$= 2\left[-\frac{1}{2(2)^2} + \frac{1}{3(2)^3} - \left(-\frac{1}{2(1)^2} + \frac{1}{3(1)^3}\right)\right]$$

$$= 2\left[-\frac{1}{8} + \frac{1}{24} - \left(-\frac{1}{2} + \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{-3+1}{24} + \frac{1}{6}\cdot\frac{4}{4}\right] = 2\left[\frac{-2}{24} + \frac{4}{24}\right] = 2\left[\frac{2}{24}\right] = \frac{1}{6}$$

Got it on 3rd try with help. :)

47 $\int_1^2 x\sqrt{x-1}\,dx$ $x=1$: $u=x-1=1-1=0$
$x=2$: $u=x-1=2-1=1$

$u = x-1$ $x=1 \to x-1=0=u$ $\Longrightarrow$
$du = dx$ $x=2 \to x-1=1=u$

$\int_0^1 x\sqrt{u}\,du$ Got an x hanging around, still.

$u=x-1 \Longrightarrow u+1=x \longrightarrow$

$\int_0^1 (u+1)u^{\frac{1}{2}}du = \int_0^1 \left(u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du$

$=\left[\frac{2}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}}\right]_0^1 = \frac{2}{5}+\frac{2}{3} = \frac{6+10}{15} = \frac{16}{15}$

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Alternate method for these definite integrals.

47 $\displaystyle\int_1^2 x\sqrt{x-1}\,dx$ $x=1$: $u=x-1=1-1=0$
$x=2$: $u=x-1=2-1=1$

$u=x-1$
$du=dx$

$\displaystyle\int x\sqrt{u}\,du$ $\quad u=x-1$, $u+1=x$ $\longrightarrow \displaystyle\int (u+1)u^{\frac{1}{2}}du$

$= \displaystyle\int \left(u^{\frac{3}{2}}+u^{\frac{1}{2}}\right)du$ "=" $\frac{2}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}} = \frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}$

$\longrightarrow \displaystyle\int_1^2 x\sqrt{x-1}\,dx = \left[\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}\right]_1^2$

$= \frac{2}{5}(2-1)^{\frac{5}{2}}+\frac{2}{3}(2-1)^{\frac{3}{2}} - (0+0)$

$x=1 \rightarrow$ 0's.

$= \frac{2}{5}+\frac{2}{3} = \frac{16}{5}$

This "sidestep" of converting limits of integration to the u-variable is handy when converting to u is a pain.

$$\int_0^{\frac{\pi}{7}} \cos x \sin(\sin x)\, dx = \int_0^{\frac{\pi}{7}} \underbrace{\sin(\sin x)}_{\sin u} \underbrace{\cos x\, dx}_{du}$$

$*$ $u = \sin x$
$du = \cos x\, dx$ $*$

$$= \int_0^{\sin \frac{\pi}{7}} \sin u\, du$$

can get ugly.

$u = \sin x$
$\sin(\frac{\pi}{7}) = \text{blah}$
$\sin(0)$

$$\sum_{i=1}^{n} i^2 = \frac{n^3 \ldots}{3}$$

$$= \Big[-\cos(\sin x)\Big]_0^{\frac{\pi}{7}}$$

$$\boxed{\int x^2\, dx = \frac{x^3}{3} + C}$$

$$\sum_{i=1}^{n} i^{17} = \frac{1}{18} n^{18} + \sim\sim$$

Lower powers than 18.

$$\frac{n^{18} + \sim\sim}{18}$$

$$= n^{18}(1 + \sim\sim)$$

positive powers of n in denominator. $\xrightarrow{n\to\infty} 0$

$$\int_0^1 x\,dx \qquad \Delta x = \frac{1-0}{n} = \frac{1}{n} = \frac{b-a}{n}$$

$$x_k = a + \Delta x \cdot k$$

$$= 0 + \frac{k}{n} = \frac{k}{n}$$

$$\approx \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} x_k \cdot \frac{1}{n} = \frac{1}{n}\sum_{k=1}^{n} \frac{k}{n} = \frac{1}{n^2}\sum_{k=1}^{n} k$$

$$= \frac{1}{n^2} \cdot \frac{n^2+n}{2} = \frac{1}{n^2} \cdot \frac{n^2(1+\frac{1}{n})}{2} = \frac{1+\frac{1}{n}}{2} \xrightarrow{n\to\infty} \boxed{\frac{1}{2}}$$

Right: $a + k\Delta x = a + \frac{b-a}{n} k \qquad x_1 = a + \frac{b-a}{n}$

Left: $a + (k-1)\Delta x = a + \frac{b-a}{n}(k-1) \qquad x_1 = a$