$\int_1^3 (x^2-1)\,dx$ by the defin

$\int^1$ 4.2 stuff

$\Delta x = \frac{3-1}{n} = \frac{2}{n} \qquad x_k = 1 + \frac{2k}{n}$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} (x_k^2 - 1) = \frac{2}{n}\sum_{k=1}^{n}\left[\left(\frac{2k}{n}+1\right)^2 - 1\right]$$

$$= \frac{2}{n}\sum_{k=1}^{n}\left[\left(\frac{4k^2}{n^2} + \frac{4k}{n} + 1\right) - 1\right]$$

$$= \frac{2}{n}\sum_{k=1}^{n}\left(\frac{4k^2}{n^2} + \frac{4k}{n}\right) = \frac{2}{n}\left[\frac{4}{n^2}\sum_{k=1}^{n} k^2 + \frac{4}{n}\sum_{k=1}^{n} k\right]$$

$$= \frac{8}{n^3}\cdot\frac{n^3 + \cdots}{3} + \frac{8}{n^2}\cdot\frac{n^2 + \cdots}{2}$$

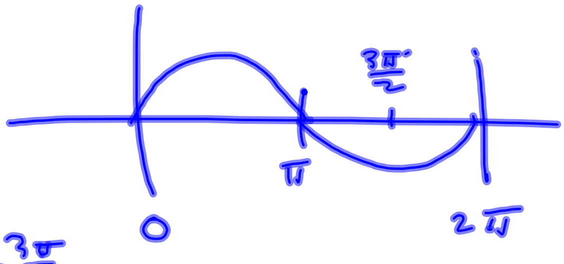
Style concepts.

$$= \frac{8}{n^3}\cdot\frac{n^3(1 + \cdots)}{3} + \frac{8}{n^2}\cdot\frac{n^2(1+\cdots)}{2}$$

$$= \frac{8}{3}(1 + \underbrace{\cdots}_{\to 0}) + 4(1 + \underbrace{\cdots}_{\to 0}) \xrightarrow{n \to \infty} \frac{8}{3} + 4 = \boxed{\frac{20}{3}}$$

§4.4 #42

$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \\ -\sin x & \text{if } \sin x < 0 \end{cases}$$



$$\int_0^{\frac{3\pi}{2}} |\sin x|\,dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{\frac{3\pi}{2}} -\sin x\,dx$$

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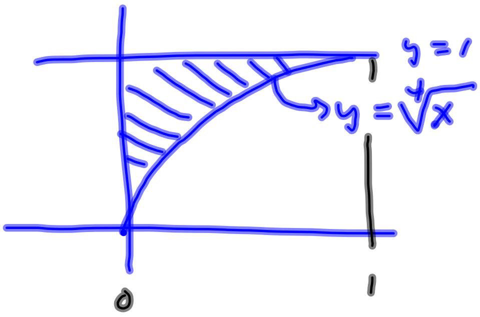
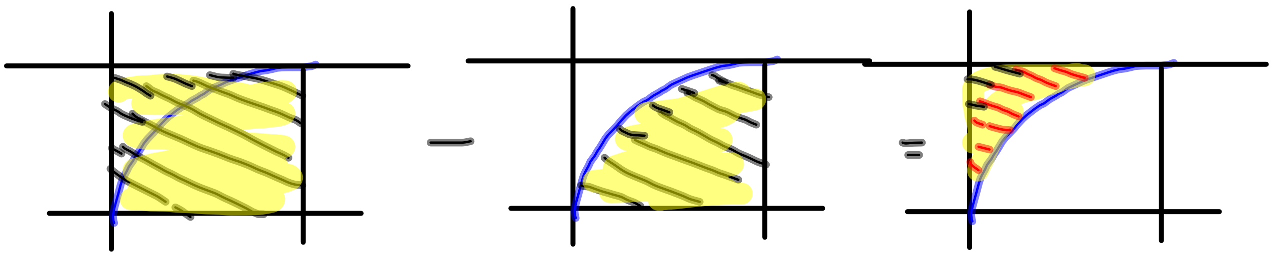
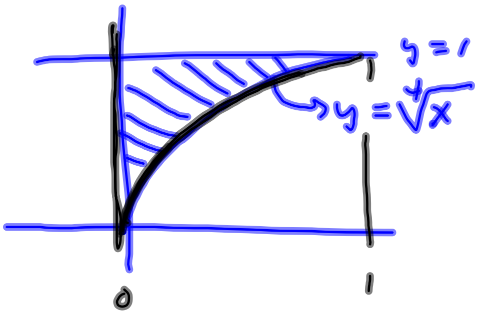


Area between $y=1$ & $y=\sqrt[4]{x}$

<u>Method</u>

$$= \int_0^1 \left(1 - x^{\frac{1}{4}}\right)dx = \int_0^1 dx - \int_0^1 x^{\frac{1}{4}}dx$$





$$\int_0^1 y^4 \, dy$$

$$y = \sqrt[4]{x} \implies y^4 = x$$

$$\int_0^1 \left(1 - x^{\frac{1}{4}}\right) dx = \int_0^1 x^4 \, dx \quad \text{weird}$$

$$\left[x - \frac{4}{5}x^{\frac{5}{4}}\right]_0^1 \overset{?}{=} \left[\frac{1}{5}x^5\right]_0^1 = \frac{1}{5}$$

$$1 - \frac{4}{5}(1)^{5/4} - \left(0 - \frac{4}{5}(0)^{\frac{5}{4}}\right)$$

$$= \frac{1}{5}$$

4.5 # 64 Show that

$$\int_0^{\pi} x\, f(\sin x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$$

use $u = \pi - x$

come back to this

§4.5 Substitution for Definite Integrals.

$$\int_{x=0}^{x=4} \frac{x}{\sqrt{1+2x}}\,dx = \frac{1}{2}\int_{x=0}^{x=4} \frac{x}{\sqrt{u}}\,du = \frac{1}{2}\int_{x=0}^{x=4} \frac{\frac{u-1}{2}}{\sqrt{u}}\,du$$

$u = 2x+1$ $\quad u-1 = 2x \Rightarrow x = \frac{u-1}{2}$

$du = 2dx$

$dx = \frac{du}{2}$

$$= \frac{1}{4}\int_{x=0}^{x=4} \frac{u-1}{u^{1/2}}\,du = \frac{1}{4}\int_{x=0}^{x=4} \left(u^{1/2} - u^{-1/2}\right) du$$

$x=0 \Rightarrow u=1$

$x=4 \Rightarrow u = 2(4)+1 = 9$

$$= \frac{1}{4}\int_{u=1}^{u=9} \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^9 = \frac{1}{4}\left[\frac{2}{3}(9)^{\frac{3}{2}} - 2(9)^{\frac{1}{2}} - \left(\frac{2}{3} - 2\right)\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}(27) - 6 + \frac{4}{3}\right]$$

$$= \frac{1}{4}\left[18 - 6 + \frac{4}{3}\right] = \frac{1}{4}\left[\frac{40}{3}\right] = \frac{10}{3}.$$

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$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx$$

You can also use the u to get the antiderivative, switch back to x-variable & then evaluate without messing with the limits of integration

$u = 2x+1 \qquad x = \frac{1}{2}(u-1)$

$du = 2dx \Longrightarrow dx = \frac{1}{2}du$

$$\int \frac{x}{\sqrt{2x+1}}\,dx = \int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\frac{1}{2}du = \frac{1}{4}\int \frac{u-1}{\sqrt{u}}\,du$$

$$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right] + C$$

$$= \frac{1}{6}u^{\frac{3}{2}} - \frac{1}{2}u^{\frac{1}{2}} + C \Longrightarrow \int_0^4 \cdots =$$

$$\left[\frac{1}{6}(2x+1)^{\frac{3}{2}} - \frac{1}{2}(2x+1)^{\frac{1}{2}}\right]_0^4$$

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