§4.2 #28, 25,

$$\int_0^1 (x^3 - 3x^2)\,dx$$

$a = 0,\ b = 1$

$\frac{b-a}{n} = \frac{1}{n} = \Delta x$

$x_k = a + k\Delta x = 0 + \frac{1}{n}k = \frac{k}{n}$

$$\int_0^1 (x^3 - 3x^2)\,dx \approx \sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f(x_k)$$

$$= \frac{1}{n}\sum_{k=1}^{n}\left(x_k^3 - 3x_k^2\right) = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^3 - \frac{3}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$$

$$= \frac{1}{n}\sum \frac{k^3}{n^3} - \frac{3}{n}\sum \frac{k^2}{n^2} =$$

$$= \frac{1}{n}\cdot\frac{1}{n^3}\sum_{k=1}^{n} k^3 - \frac{3}{n}\cdot\frac{1}{n^2}\sum_{k=1}^{n} k^2 = \frac{1}{n^4}\cdot\frac{n^4 + \cdots}{4} - \frac{3}{n^3}\,\frac{n^3 + \cdots}{3}$$

$$= \frac{n^4 + \cdots}{4n^4} - \frac{3n^3 + \cdots}{3n^3} \xrightarrow{n \to \infty} \frac{1}{4} - 1 = \boxed{-\frac{3}{4} = \int_0^1 (x^3 - 3x^2)\,dx}$$

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$$\int_a^b x^2\,dx$$

$$\sum f(x)\Delta x = \sum \left(a + \frac{b-a}{n}k\right)^2\left(\frac{b-a}{n}\right)$$

$$\sum_{k=1}^{n}\left[\left(a^2 + 2\,\frac{b-a}{n}ka + \left(\frac{b-a}{n}k\right)^2\right)\left(\frac{b-a}{n}\right)\right]$$

$$= \frac{b-a}{n}\sum_{k=1}^{n}\left(a^2 + 2a\frac{b-a}{n}k + \frac{b^2-2ab+a^2}{n^2}k^2\right)$$

$$= \frac{b-a}{n}a^2\sum_{k=1}^{n}1 + \frac{b-a}{n}\cdot 2a\frac{b-a}{n}\sum_{k=1}^{n}k + \boxed{\frac{b-a}{n}\cdot\frac{b^2-2ab+a^2}{n^2}}\sum_{k=1}^{n}k^2$$

$$= \frac{b-a}{n}a^2\cdot n + 2a\left(\frac{(b-a)^2}{n^2}\right)\cdot\frac{n^2+\cdots}{2} + \frac{(b-a)^3}{n^3}\cdot\frac{n^3+\cdots}{3}$$

$$= a^2(b-a) + 2a\frac{(b-a)^2}{2n^2}\cdot(n^2+\cdots) + \frac{(b-a)^3}{3n^3}\cdot(n^3+\cdots)$$

$$\xrightarrow{n\to\infty} a^2(b-a) + a(b-a)^2 + \frac{(b-a)^3}{3}$$

$$(b-a)\left[a^2 + a(b-a) + \frac{(b-a)^2}{3}\right]$$

$$= (b-a)\left[a^2 + ab - a^2 + \frac{1}{3}\left[b^2-2ab+a^2\right]\right]$$

$$= (b-a)\left[ab + \frac{1}{3}b^2 - \frac{2}{3}ab + \frac{1}{3}a^2\right]$$

$$= (b-a)\left(\frac{1}{3}ab + \frac{1}{3}b^2 + \frac{1}{3}a^2\right)$$

$$= \frac{1}{3}\underline{(b-a)(b^2+ab+a^2)} =$$

$$b^3 + ab^2 + a^2b$$
$$- ab^2 - a^2b - a^3$$
$$= b^3 - a^3$$

$$= \frac{1}{3}(b^3 - a^3)$$

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§4.3 Like #29

$$\int_1^2 \frac{x^2-\sqrt{x}}{\sqrt{x}}\,dx = \boxed{\int_1^2 \left(x^{3/2} - 1\right)} dx \qquad \frac{x^2}{\sqrt{x}} = x^{2-\frac{1}{2}} \qquad \frac{\sqrt{x}}{\sqrt{x}} = 1$$

$$= \left[\frac{2}{5} x^{5/2} - x\right]_1^2 \qquad 2^{\frac{5}{2}} = 2^{\frac{4}{2}+\frac{1}{2}} = 2^2 \cdot 2^{\frac{1}{2}} = 4\sqrt{2}$$

$$2^{5/2} = \left(2^{\frac{1}{2}}\right)^5$$

$$= \frac{2}{5}(2)^{\frac{5}{2}} - 2 - \left[\frac{2}{5}(1)^{\frac{5}{2}} - 1\right] \qquad = \underbrace{2^{\frac{1}{2}}2^{\frac{1}{2}}}_{2} \cdot \underbrace{2^{\frac{1}{2}}2^{\frac{1}{2}}}_{2} \cdot \underbrace{2^{\frac{1}{2}}}_{\sqrt{2}}$$

$$= \frac{8\sqrt{2}}{5} - 2 - \frac{2}{5} + 1 = \frac{8\sqrt{2} - 10 - 2 + 5}{5}$$

$$= \frac{8\sqrt{2} - 7}{5}$$

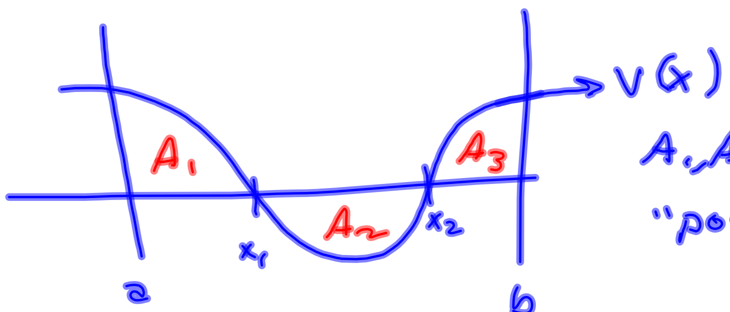
Displacement = Net change

Distance = Gross Change

in the context of $\int_a^b$ velocity

$$\text{Displacement} = \int_a^b f(x)\,dx$$

$$\text{Distance} = \int_a^b |f(x)|\,dx$$



A_1, A_2, A_3 are "positive" areas.

$$\text{Displacement} = A_1 - A_2 + A_3$$
$$= \int_a^b v(x)\,dx$$

$$\text{Distance} = \int_a^b |v(x)|\,dx = \int_a^{x_1} v(x)\,dx - \int_{x_1}^{x_2} v(x)\,dx + \int_{x_2}^{b} v(x)\,dx$$

§4.4# 5, 11, 19, 27, 33, $\boxed{\text{39 - 42 ALL}}$, 46, 48, 49, 55, 57

$$\frac{d}{dx}\left[\sin(x^2+7x)\right] = \cos(x^2+7x)\cdot(2x+7)$$

§4.5: $\int \cos(x^2+7x)\,dx = ???$

$$\int \cos(x^2+7x)\cdot(2x+7)\,dx = \sin(x^2+7x)+C$$

Let $u = x^2+7x$

Then $\frac{du}{dx} = (2x+7)$

$$\int \cos(\underbrace{x^2+7x}_{u})\underbrace{(2x+7)\,dx}_{du} = \int \cos(u)\cdot\frac{du}{dx}\cdot dx$$

$$= \int \cos(u)\,du$$

$$= \sin u + C$$

$$= \sin(x^2+7x) + C$$

$$\int (x^5-7x^4+2)^{11}\,dx = ??$$

$$\int (x^5-7x^4+2)^{11}(5x^4-28x^3)\,dx = \int u^{11}\,du = \frac{u^{12}}{12}+C$$

$u = x^5-7x^4+2$

$$\boxed{du = (5x^4-28x^3)\,dx}$$

$$= \frac{(x^5-7x^4+2)^{12}}{12}+C$$

$\frac{du}{dx} = 5x^4-28x^3 \implies$

$du = (5x^4-28x^3)\,dx \implies$

$$\boxed{\frac{du}{5x^4-28x^3} = dx}$$ makes mechanics easier?

Then $\int (x^5-7x^4+2)^{11}(5x^4-28x^3)\,dx$

$$= \int u^{11}(5x^4-28x^3)\left(\frac{du}{5x^4-28x^3}\right)$$

$$= \int u^{11}\,du$$

Method has advantages for handling constant multiples in the du part.

4

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$\int \sqrt{3x-7}\,dx$ $\int \cos(3x-7)\,dx$

$u = 3x-7 \Longrightarrow$

$du = 3\,dx$

$\frac{du}{3} = dx$

My students like this

$$\int \sqrt{u} \cdot \frac{du}{3} = \frac{1}{3}\int u^{\frac{1}{2}}\,du$$

The way $\underline{I}$ see it:

$\int \sqrt{3x-7}\,dx$

$u = 3x-7$

$du = 3dx$

Make the 3dx

$$= \frac{1}{3}\int \sqrt{3x-7} \cdot 3dx$$

Rectify the changes.