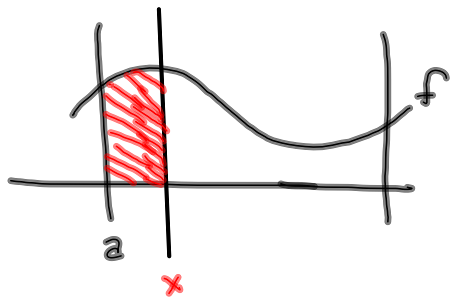
§4.3 FTC
Fundamental Theorem of Calculus.

Think of $\int_a^b f(x)dx$ as an area

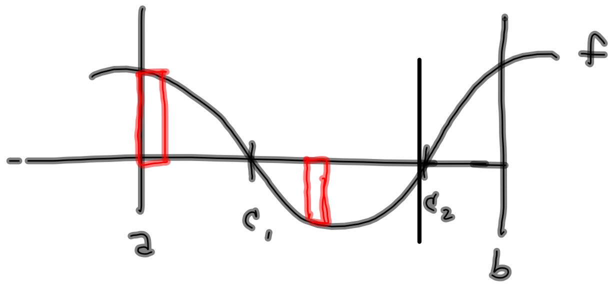
$$\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(\xi)d\xi$$

$$g(x) = \int_a^x f(t)dt$$

If $f(x) \geq 0$, then $g(x)$ is nondecreasing
$f(x) > 0$ " " " increasing.



$g(x)$ is area from a to x under the curve $f(x)$.



$\int_a^b f(t)dt$ gives "signed area."

$g(x)$ has a local max: $x = c_1$

$$g(x) = \int_a^x f(t)dt$$

$g(x)$ has local min @ $x = c_2$
g is increasing on (a, c_1)
" " decreasing on (c_1, c_2)

FTCI

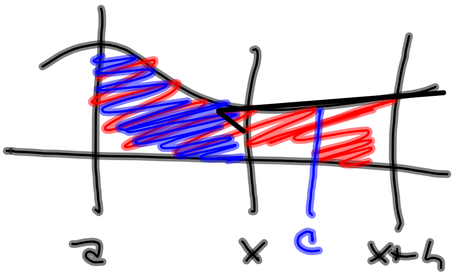
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

The derivative of the integral is the integrand.

$$g(x) = \int_a^x f(t)\,dt \longrightarrow$$

$$\frac{g(x+h)-g(x)}{h} = \frac{1}{h}\left[g(x+h)-g(x)\right]$$

$$= \frac{1}{h}\left[\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt\right]$$



$$= \frac{1}{h}\left[\int_x^{x+h} f(t)\,dt\right]$$

Separate idea.

$$\frac{f(x+h)-f(x)}{h} = f'(c) \text{ for some } c \text{ between } x \text{ and } x+h$$

The SAME sort of thing is happening up above

$$\frac{\int_x^{x+h} f(t)\,dt}{h} = f(x^*) \text{ for some } x^* \text{ between } x \text{ \& } x+h.$$

when $h \to 0$, the $x^* \longrightarrow x$ and we get $\frac{d}{dx}\int^x f(t)\,dt = f(x)$.

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CHAIN RULE VERSION

$$\frac{d}{dx}\left[g(x^2-5x)\right] = \frac{dg}{d(x^2-5x)} \cdot \frac{d(x^2-5x)}{dx}$$

$$\frac{d}{dx}\left[\sin(x^2-5x)\right] = \cos(x^2-5x)\cdot(2x-5)$$

$$\frac{d}{dx}\int_a^{x^2-5x} f(t)\,dt = \left(f(x^2-5x)\right)(2x-5)$$

I'm leaving out some details.
Like hypotheses on f
conditions on f in the hypotheses.
(Continuity).

That was FTC I

Now FTC II

If $F'(x) = f(x)$, i.e., $\int f(x)\,dx = F(x) + C$

Then $\int_a^b f(x)\,dx = F(b) - F(a)$

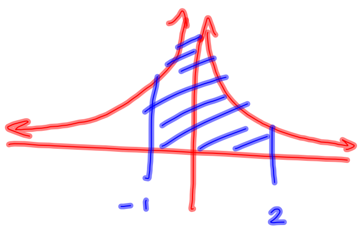
$\boxed{E}$ $\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$

$\boxed{NE}$ $\int_0^2 \frac{1}{x-1}\,dx$ Newp

$$\int_{-1}^{2} \frac{1}{x^2}\,dx = \int_{-1}^{2} x^{-2}\,dx = \left[\frac{x^{-1}}{-1}\right]_{-1}^{2} = \left[-\frac{1}{x}\right]_{-1}^{2}$$

$$= -\frac{1}{2} - \left(-\frac{1}{-1}\right) = -\frac{3}{2}$$

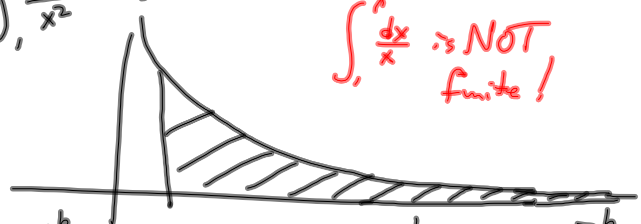
Totally Bogus: $\frac{1}{x^2}$ isn't cont's on $[-1,2]$.



$\int_{-1}^{2} \frac{dx}{x^{1/3}}$ is finite, but doesn't exist as a "proper Riemann integral"

$\int_1^\infty \frac{dx}{x^2}$ is finite

$\int_1^x \frac{dx}{x}$ is NOT finite!



$$\lim_{b\to\infty} \int_1^b x^{-2}\,dx = \lim_{b\to\infty} \left[-x^{-1}\right]_1^b = \lim_{b\to\infty} \left[-\frac{1}{x}\right]_1^b$$

$$\lim_{b\to\infty} \left[-\frac{1}{b} - \left(-\frac{1}{1}\right)\right] = 1$$

§4.3 #s 2, 3, 7, 25, 29, 39*, 47, 49, 53!

$$\int_a^b f = -\int_b^a f$$

dx's are "negative" marching from b, left to a.

$$\frac{d}{dx}\int_x^a f = ?$$

$$\int_a^a f = 0$$

$$\frac{d}{dx}\int_a^{u(x)} f(t)\,dt$$

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