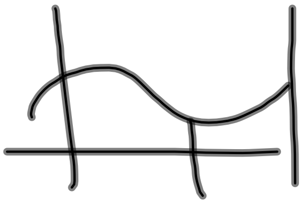
Recall: Right endpoint Riemann Sum

$\Delta x = \frac{b-a}{n}$, $x_k = a + k\Delta x = a + \frac{b-a}{n}k$



Area $\approx \sum_{k=1}^{n} f(x_k)\Delta x$

$= \Delta x \sum_{k=1}^{n} f(x_k)$, since $\Delta x = \frac{b-a}{n}$ is same for each term.

$\xrightarrow{n\to\infty} \int_a^b f(x)\,dx$

Left endpoint Riemann Sum

$x_k = a + (k-1)\Delta x = a + \frac{b-a}{n}(k-1)$

WE ALWAYS use equal-width rectangles.

More generally, we say $\|P\| =$ mesh of the partition

$\|P\| = \|\Delta x\| \longrightarrow 0$

$\|\Delta x\| = \max\{\Delta x_k\}$

Formally

$$\lim_{\|\Delta x\|\to 0} \sum_{k=1}^{n} f(x_k)\Delta x_k = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

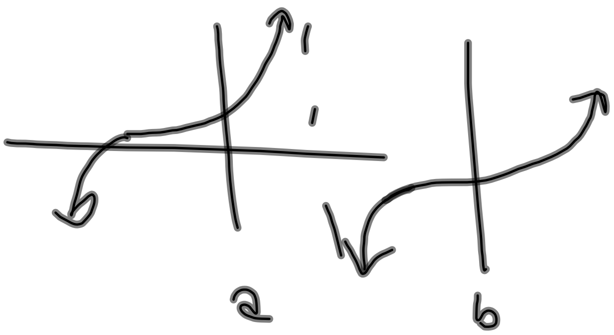
with equal widths, $n\to\infty$ guarantees $\|\Delta x\| \longrightarrow 0$.

§4.2 cont'd

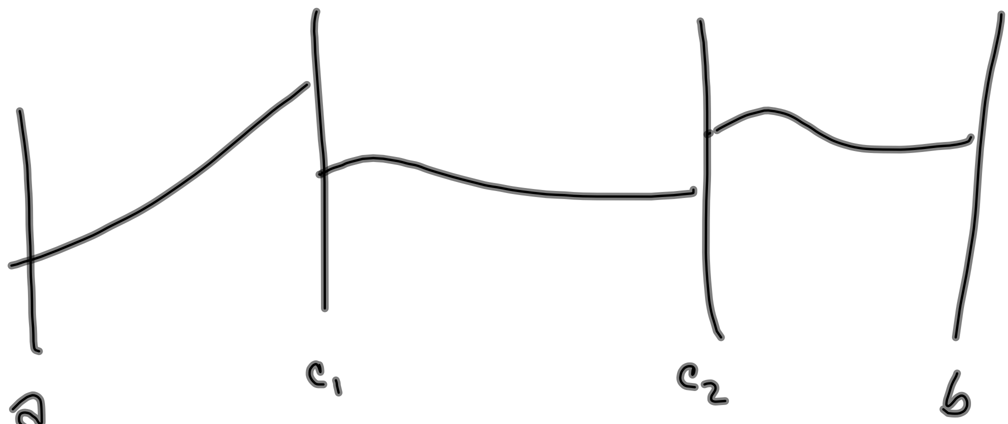
If f is cont^s on $[a,b]$ (or only has finitely many "jump" (finite) discontinuities), then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

is our definition



Infinite Discontinuity & all bets are off.



$$\int_a^b f = \int_a^{c_1} f + \int_{c_1}^{c_2} f + \int_{c_2}^{b} f$$

Finite # of discontinuities.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = 1+2+3+\dots+n$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = 1^2+2^2+3^2+\dots+n^2$$

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2 = 1^3+2^3+3^3+\dots+n^3$$

For our purposes, we're looking at these things in $\lim_{n\to\infty} \sum(\text{stuff})$ and

ONLY THE BIG STUFF MATTERS.

$$\frac{n(n+1)}{2} = \frac{n^2+n}{2} = \boxed{\frac{n^2+\sim}{2}} \qquad \int x\,dx = \frac{x^2}{2}+C$$

$$\frac{n(n+1)(2n+1)}{6} = \frac{2n^3+\sim}{6} = \boxed{\frac{n^3+\sim}{3}} \qquad \int x^2\,dx = \frac{x^3}{3}+C$$

$$\left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \frac{n^2(n+1)(n+1)}{4} = \boxed{\frac{n^4+\sim}{4}} \qquad \int x^3\,dx = \frac{x^4}{4}+C$$

$$\sum k^2 = \frac{n^3+\sim}{3}$$

Misc. $\sum$ props

$$\sum_{k=1}^{n} 7 = \underbrace{7+7+7+\cdots+7}_{n \text{ of 'em}} = 7n$$

$$\sum_{k=1}^{n} 7 = 7\sum_{k=1}^{n} 1 = 7[\underbrace{1+1+\cdots+1}_{n \text{ of 'em}}] = 7n$$

$$\sum_{k=1}^{n} 7f(x) = 7\sum_{k=1}^{n} f(x)$$

$$\sum_{k=1}^{n} (a_k+b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$\sum$ is linear operator

$$\left(\sum_{k=1}^{n} 0 = 0\right)$$

Commutativity

$$a_1+b_1+a_2+b_2+\cdots+a_n+b_n$$
$$= a_1+a_2+\cdots+a_n + b_1+b_2+\cdots+b_n$$
$$= \sum a_k + \sum b_k$$

Comparison Properties Pg 305

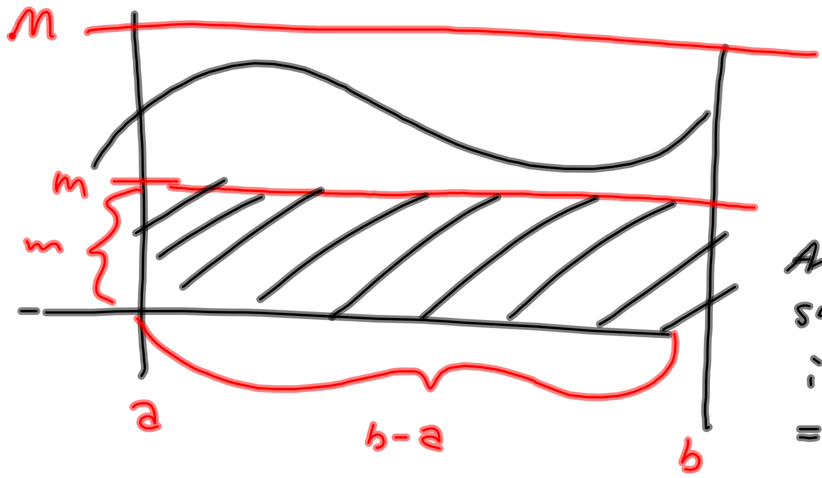
6 $f \geq 0$ on $[a,b]$ $\Longrightarrow \int_a^b f \geq 0$ Rectangles with positive height.

7 $f \geq g$ on $[a,b]$ $\Longrightarrow \int_a^b f \geq \int_a^b g$

"Area under the taller function is greater."

8 Floor & Ceiling.

$m \leq f < M \Longrightarrow m(b-a) \leq \int_a^b f \leq M(b-a)$



Area of the smaller rectangle is h·w $= m(b-a)$

Find the area under $x^2+2x+5 = f(x)$ on $[0,3]$ as the LIMIT OF A RIEMANN SUM.

$a=0,\ b=3 \quad \Delta x = \frac{b-a}{n} = \frac{3}{n}$

$x_k = a+k\Delta x = 0+\frac{3}{n}k = \frac{3k}{n}$

x^2+2x+5 is big & messy. We break it into pieces & add pieces.

$$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2+2\left(\frac{3k}{n}\right)+5\right)$$

$$= \frac{3}{n}\left[\overset{A}{\sum_{k=1}^{n}\left(\frac{3k}{n}\right)^2} + \overset{B}{\sum_{k=1}^{n}2\left(\frac{3k}{n}\right)} + \overset{C}{\sum_{k=1}^{n}5}\right]$$

A: $\frac{3}{n}\sum_{k=1}^{n}\left(\frac{3k}{n}\right)^2 = \frac{3}{n}\sum_{k=1}^{n}\frac{9k^2}{n^2} = \frac{3}{n}\left[\frac{9}{n^2}1^2+\frac{9}{n^2}2^2+\frac{9}{n^2}3^2+\cdots+\frac{9}{n^2}n^2\right]$

$$= \frac{3}{n}\cdot\frac{9}{n^2}\sum_{k=1}^{n}k^2 = \frac{27}{n^3}\cdot\frac{n^3+\cdots}{3}$$

$$\xrightarrow{n\to\infty} \frac{27}{3} = 9$$

B $\frac{3}{n}\sum_{k=1}^{n}2\left(\frac{3k}{n}\right) = \frac{18}{n^2}\sum k = \frac{18}{n^2}\cdot\frac{n^2+\cdots}{2} \xrightarrow{n\to\infty} \frac{18}{2} = 9$

C $\frac{3}{n}\sum_{k=1}^{n}5 = \frac{3}{n}\cdot 5n = 15$

Meg

$A+B+C = 9+9+15 = 33$

$$\int_0^3 (x^2+2x+5)\,dx = \left[\frac{x^3}{3}+x^2+5x\right]_0^3 = 9+9+15 - \left[\frac{0^3}{3}+0^2+5(0)\right]$$

$= 33.$