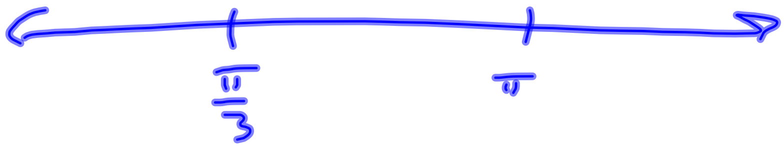


$1+\cos x$ $1-\cos x$

$1-\sin x$

$\cos x - 1$ exceptions to sign changes at x-intercepts.

Bulletproof: Test values.

$$1+2+3+4+\cdots+(n-1)+n=\sum_{k=1}^{n} k=\frac{n(n+1)}{2}$$

$$\sum_{k=1}^{100} k=\frac{100(101)}{2}=5050$$

Nope. ~~1+99, 2+98, ⋮, 49+51~~, 50+50

1+100
2+99
3+98
⋮
49+52
50+51

$$50(101)=\frac{100(101)}{2}$$

Principle of Mathematical Induction.

Want something* to hold for all natural numbers $= \mathbb{N} = \{1,2,3,4,\ldots\}$.

* Some formula.

Let S = the set of all natural #s such that the statement holds.

1st prove that $1 \in S$ (i.e., prove $S \neq \emptyset$)

2nd prove that any time $k \in S$, then $k+1 \in S$

Claim: $P(n)$: $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

Proof Let S = set of all n such $P(n)$ holds.

$\displaystyle\sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2}$, so $1 \in S \neq \emptyset$.

Assume $k \geq 1$ is in S.

Then $\displaystyle\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ and

$$\sum_{i=1}^{k+1} i = \underbrace{1+2+3+\cdots+k}_{\frac{k(k+1)}{2}} + (k+1)$$

$$+\ k+1 \qquad (k+1)((k+1)+1)$$

$$= \frac{k^2+k}{2}+\frac{2k+2}{2}=\frac{k^2+3k+2}{2}=\frac{(k+1)(k+2)}{2}$$

Trick is seeing what $P(k+1)$ must look like:

$P(k)$: $\displaystyle\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ Bootstrap from this

to $P(k+1)$: $\displaystyle\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

$P(n)$: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Pf Let $S = \{ n \in \mathbb{N} \mid P(n) \text{ holds} \}$

Since $\sum_{i=1}^{1} i^2 = 1 = \frac{1(1+1)(2+1)}{6} = \frac{1(2)(3)}{6} = 1$ ✓

we have $1 \in S \neq \emptyset$. Assume $k \in S$. we show

$k+1 \in S$:

$$\sum_{i=1}^{k+1} i^2 = 1 + 2^2 + \cdots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)[2k^2 + k + 6k + 6]}{6} = \frac{(k+1)[2k^2 + 7k + 6]}{6}$$

$$= \frac{(k+1)[2k^2 + 3k + 4k + 6]}{6} = \frac{(k+1)[k(2k+3) + 2(2k+3)]}{6}$$

$$= \frac{(k+1)(2k+3)(k+2)}{6} = ?$$

Goal $= \frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$

$\Rightarrow k+1 \in S$

$\therefore S = \mathbb{N}$, by PMI. ∎

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

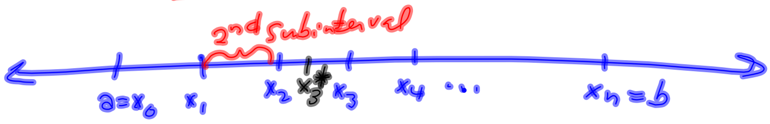
$$\boxed{\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2}$$

Prove for 1 full Homework assignment.

§4.1 #s 4

§4.2 #s 17, 19*, 21–25, 27, 28, 33a, 37, 41

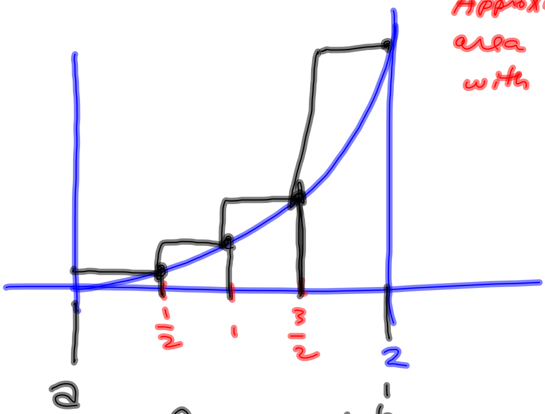
x_i^* is ANY # in $[x_{i-1}, x_i]$ = the i^{th} subinterval.



§4.1 Riemann Sums. Focus on Right endpoint.
Except #4b, §4.1.

Consider $f(x)=x^2$ on $[0,2]$

Approximate the area under $f(x)$ with 4 rectangles



Area ≈ area of the rectangles.

$= h_1w_1 + h_2w_2 + h_3w_3 + h_4w_4$

$= f(x_1)\cdot\Delta x + f(x_2)\Delta x + f(x_3)\Delta x_3 + f(x_4)\Delta x_4$

$= \sum_{k=1}^{4} f(x_k)\Delta x$, where $f(x)=x^2$

$x_k = a + k\cdot\Delta x$

$= 0 + k\cdot\frac{1}{2}$

$= a + k\cdot\frac{b-a}{n}$

$= 0 + k\cdot\frac{2-0}{4}$

$= \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + (1)^2\left(\frac{1}{2}\right) + \left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right) + (2)^2\left(\frac{1}{2}\right)$

$= \left(\frac{1}{2}\right)\left[\frac{1}{4} + 1 + \frac{9}{4} + 4\right] = \frac{1}{2}\left[\frac{1+4+9+16}{4}\right]$

$= \frac{30}{8} = \boxed{\frac{15}{4}}$

Our Goal: Take $n \longrightarrow \infty$.

$[a,b] = [0,2]$, $f(x) = x^2$ $\quad \Delta x = \frac{b-a}{n} = \frac{2}{n}$

$x_k = a + k \cdot \frac{2}{n}$

$= 0 + \frac{2k}{n} = \frac{2k}{n}$

$$\sum_{k=1}^{n} f(x_k)\Delta x =$$

$$= \sum_{k=1}^{n} \left(\frac{2k}{n}\right)^2 \cdot \frac{2}{n} = \sum_{k=1}^{n} \frac{4k^2}{n^2} \cdot \frac{2}{n} = \frac{8}{n^3}\sum_{k=1}^{n} k^2$$

$$= \frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{8}{n^3}\left[\frac{2n^3 + \text{smaller degree}}{6}\right] \xrightarrow{n\to\infty}$$

$$\frac{16n^3}{6n^3} = \frac{16}{6} = \frac{8}{3}$$

Oct 23-12:57 PM