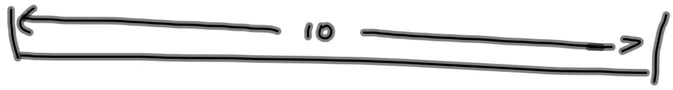
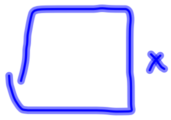
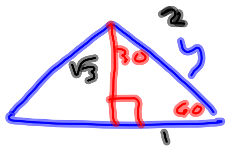
§3.7 #35



$0 \le x \le 10$

y





$\frac{h}{y} = \sin 60° = \frac{\sqrt{3}}{2}$

$h = \frac{\sqrt{3}}{2} y$

Maximize area enclosed

$4x + 3y = 10$

$\text{Area} = x^2 + \frac{1}{2} y \cdot \frac{\sqrt{3}}{2} y = x^2 + \frac{\sqrt{3}}{4} y^2$

$3y = 10 - 4x$

$y = \frac{1}{3}(10 - 4x)$

$A(x) = x^2 + \frac{\sqrt{3}}{4} \cdot \left(\frac{1}{3}(10-4x)\right)^2$ → Yes

$= x^2 + \frac{\sqrt{3}}{12}(10-4x)^2$ → No, 4·9, not 4·3

$A'(x) = 2x + \frac{\sqrt{3}}{\cancel{12}\ \cancel{6}\ 3} \cdot \cancel{2}\,(10-4x)(-\cancel{4}\ 2)$

$= 2x - \frac{2\sqrt{3}}{3}(10-4x)$

$= 2x - \frac{20\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} x$

$= \frac{6 + 8\sqrt{3}}{3} x - \frac{20\sqrt{3}}{3} \overset{\text{SET}}{=} 0$

$\Rightarrow (6 + 8\sqrt{3}) x = 20\sqrt{3}$

$x = \frac{20\sqrt{3}}{6 + 8\sqrt{3}} = \frac{10\sqrt{3}}{3 + 4\sqrt{3}}$ → 9

$eqn1 := 4 \cdot x + 3 \cdot y = 10$

$$4\,x + 3\,y = 10$$

$solve(eqn1, y)$

$$-\frac{4}{3}\,x + \frac{10}{3}$$

$f := x \rightarrow x^2 + \frac{\mathrm{sqrt}(3)}{4} \cdot \left(-\frac{4}{3}\,x + \frac{10}{3}\right)^2$

$$x \rightarrow x^2 + \frac{1}{4}\sqrt{3}\left(-\frac{4}{3}\,x + \frac{10}{3}\right)^2$$

$fp := \mathrm{D}(f)$

$$x \rightarrow 2\,x - \frac{2}{3}\sqrt{3}\left(-\frac{4}{3}\,x + \frac{10}{3}\right)$$

$solve(fp(x) = 0)$

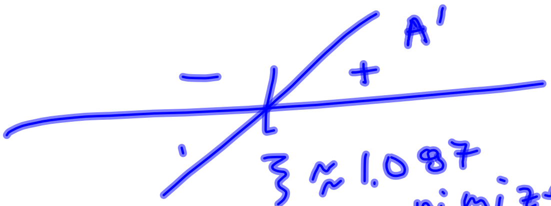
$$\frac{10\sqrt{3}}{9 + 4\sqrt{3}} = \xi$$

$expand\left(2\,x - \frac{2}{3}\sqrt{3}\left(-\frac{4}{3}\,x + \frac{10}{3}\right)\right)$

$$2\,x + \frac{8}{9}\sqrt{3}\,x - \frac{20}{9}\sqrt{3}$$

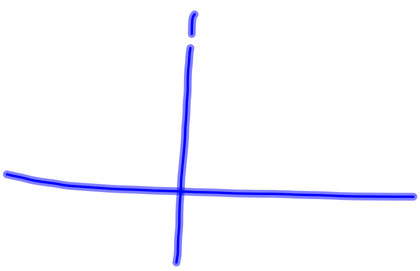
So $A(x)$ is a parabola, opening up.

$A'(x) = fp(x)$ is a line



So ξ gives a minimum.

$\xi \approx 1.087$ minimizes area.



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§3.8 #s 5-8 ALL, 11, 13

§3.9 #s 1-17, 21, 27, 41

§3.9

An infinite # of functions have a derivative $f'(x) = x^2 + 5x + 2$

They're all the same, except for (up to) a constant.

$f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 2x + 7$ is one.

$f'(x) = x^2 + 5x + 2$. See?

$f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 2x - 513{,}283$ is another.

$f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 2x + C$ represents ALL anti-derivatives.

Knowing C would uniquely determine f.

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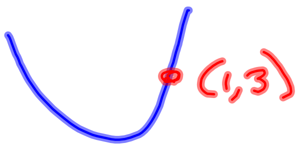
Solve the Differential Equation

$y' = 3x-2$; $y(1) = 3$.

$y = \frac{3}{2}x^2 - 2x + C$ is general solution

$y(1) = 3$:



$\frac{3}{2} - 2(1) + C = 3$

$\frac{3}{2} - \frac{4}{2} + C = \frac{6}{2}$

$C = \frac{7}{2}$ Ryan.

$\therefore$ $\boxed{y = \frac{3}{2}x^2 - 2x + \frac{7}{2}}$ is the ONLY member of this family of antiderivatives that passes thru (1,3)

$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x$

$\int \sec x \tan x \, dx = \sec x + \underbrace{C}_{\text{Arbitrary "constant of integration."}}$

The two faces: Derivative and Antiderivative