§3.7 # 20

Closest point on $\sqrt{x}$ to $(3,0)$

D = Distance : $\sqrt{(x-3)^2+(\sqrt{x}-0)^2} \geq 0$

$(x,\sqrt{x})$ to $(3,0)$

$0 \leq D$ is min when D^2 is min

minimize $(x-3)^2+(\sqrt{x})^2 = D^2$

$x^2-6x+9+x = x^2-5x+9 = y$

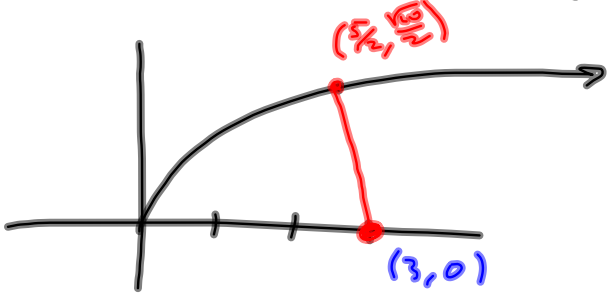
$(\sqrt{x})^2 = x \quad (x \geq 0$ already$)$

$\sqrt{x^2} = |x|$

$y' = 2x-5 \overset{SET}{=} 0 \Rightarrow 2x=5 \Rightarrow x=\frac{5}{2}$

$\Rightarrow y = \sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

$(x,y) = \left(\frac{5}{2}, \frac{\sqrt{10}}{2}\right)$

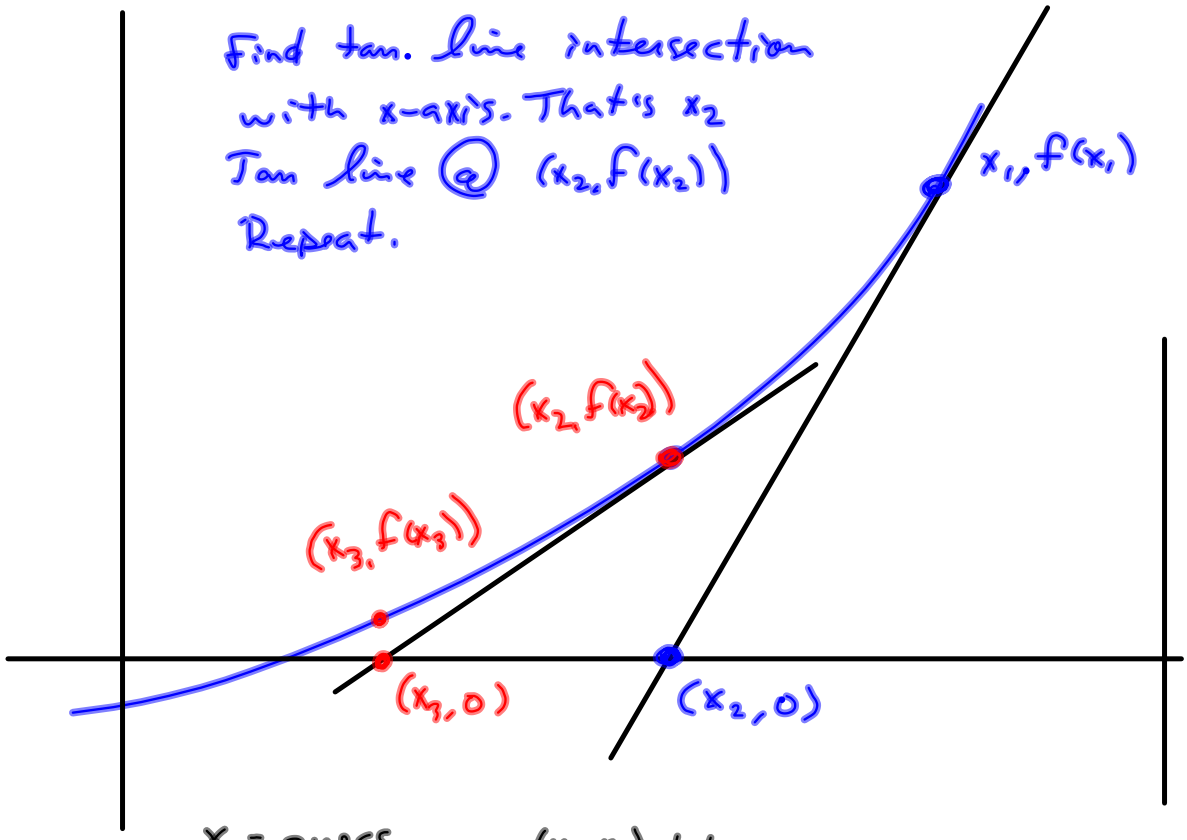


§3.8 Newton's Method

Find tan. line intersection with x-axis. That's x_2

Tan line @ $(x_2, f(x_2))$

Repeat.



x_1 = guess. $m(x - x_1) + y_1$

x_2: $y = f'(x_1)(x - x_1) + f(x_1)$ set $= 0$

$f'(x_1)x - f'(x_1)x_1 + f(x_1) = 0$

(x → x_2)

$f'(x_1)x_2 = f'(x_1)x_1 - f(x_1)$

$x_2 = \dfrac{f'(x_1)x_1 - f(x_1)}{f'(x_1)}$

$= x_1 - \dfrac{f(x_1)}{f'(x_1)}$

Use Newton's method to find zero of $f(x) = x^2 - 2x - 1$

Guess: $x_1 = 1.5$

§3.8 # 26 Find all solns.

$$\cos(x^2 - x) = x^4$$

$$f(x) = \cos(x^2 - x) - x^4 = 0$$

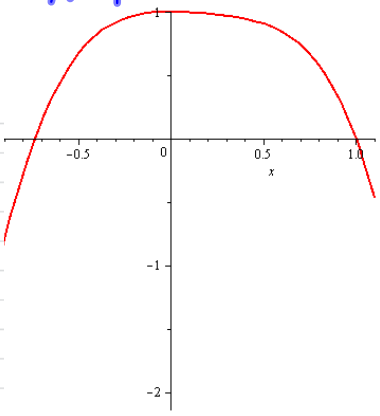
$$f'(x) = (-\sin(x^2 - x))(2x - 1) - 4x^3$$

$x_1 = 0$

Spreadsheet work:

xk	f(xk)	f'(xk)		xk	f(xk)	f'(xk)
-0.5	0.669189	1.863278		0.5	0.906412	-0.5
-0.85915	-0.57132	5.253992		2.312825	-29.608	-49.8676
-0.75041	-0.06264	4.108745		1.719093	-8.40529	-22.6246
-0.73516	-0.00119	3.952785		1.347582	-2.40548	-10.554
-0.73486	-4.6E-07	3.94972		1.119661	-0.58058	-5.78016
-0.73486	-6.9E-14	3.949719		1.019218	-0.07931	-4.25541
-0.73486	0	3.949719		1.000581	-0.00233	-4.00756
-0.73486	0	3.949719		1.000001	-2.2E-06	-4.00001
-0.73486	0	3.949719		1	-2E-12	-4
-0.73486	0	3.949719		1	0	-4
-0.73486	0	3.949719		1	0	-4
-0.73486	0	3.949719		1	0	-4
-0.73486	0	3.949719		1	0	-4

Maple Plot



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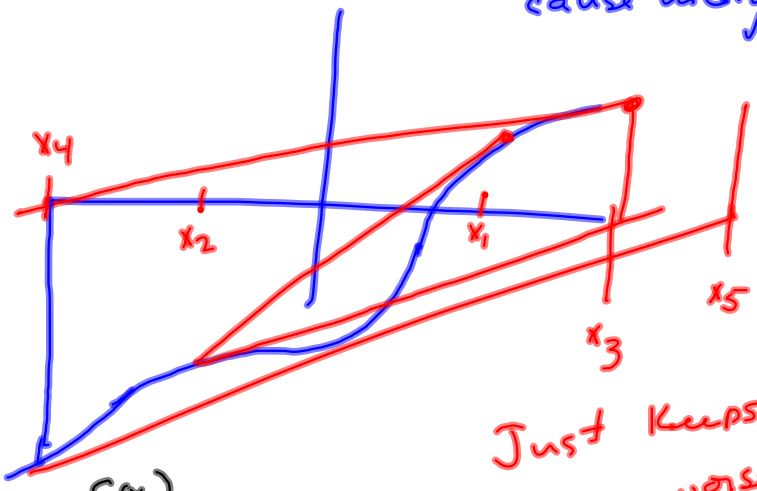
Formulas in Excel.

	A	B	C
1	xk	f(xk)	f'(xk)
2	-0.5	=COS(A2^2-A2)-A2^4	=(2*A2-1)*(-SIN(A2^2-A2))-4*A2^3
3	=A2-B2/C2	=COS(A3^2-A3)-A3^4	=(2*A3-1)*(-SIN(A3^2-A3))-4*A3^3
4	=A3-B3/C3	=COS(A4^2-A4)-A4^4	=(2*A4-1)*(-SIN(A4^2-A4))-4*A4^3
5	=A4-B4/C4	=COS(A5^2-A5)-A5^4	=(2*A5-1)*(-SIN(A5^2-A5))-4*A5^3
6	=A5-B5/C5	=COS(A6^2-A6)-A6^4	=(2*A6-1)*(-SIN(A6^2-A6))-4*A6^3
7	=A6-B6/C6	=COS(A7^2-A7)-A7^4	=(2*A7-1)*(-SIN(A7^2-A7))-4*A7^3
8	=A7-B7/C7	=COS(A8^2-A8)-A8^4	=(2*A8-1)*(-SIN(A8^2-A8))-4*A8^3
9	=A8-B8/C8	=COS(A9^2-A9)-A9^4	=(2*A9-1)*(-SIN(A9^2-A9))-4*A9^3
10	=A9-B9/C9	=COS(A10^2-A10)-A10^4	=(2*A10-1)*(-SIN(A10^2-A10))-4*A10^3
11	=A10-B10/C10	=COS(A11^2-A11)-A11^4	=(2*A11-1)*(-SIN(A11^2-A11))-4*A11^3
12	=A11-B11/C11	=COS(A12^2-A12)-A12^4	=(2*A12-1)*(-SIN(A12^2-A12))-4*A12^3
13	=A12-B12/C12	=COS(A13^2-A13)-A13^4	=(2*A13-1)*(-SIN(A13^2-A13))-4*A13^3
14	=A13-B13/C13	=COS(A14^2-A14)-A14^4	=(2*A14-1)*(-SIN(A14^2-A14))-4*A14^3

Newton's Fails : Concave Down can cause indigestion.



Just keeps getting worse.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -\sqrt{-x} & \text{if } x < 0 \end{cases}$$