§3.5 #33

$y = \sin^3 x$

$\frac{\pi}{2} + \frac{\pi}{2} - .955$

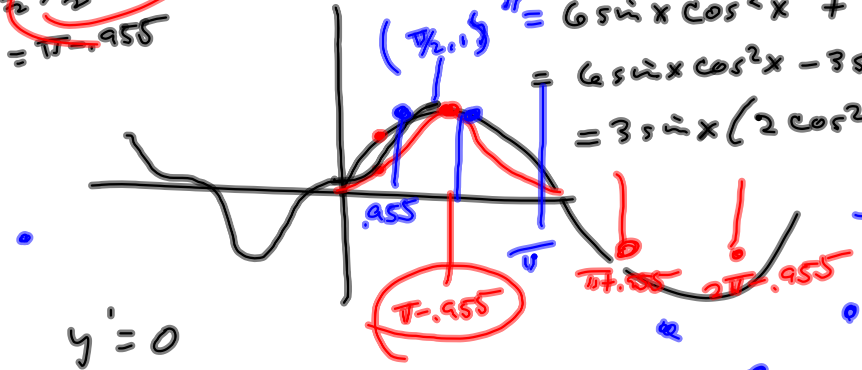
$= \pi - .955$

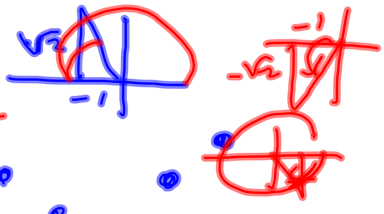
$y' = 3\sin^2 x \cos x$

$y'' = 6\sin x \cos^2 x + 3\sin^2 x(-\sin x)$

$= 6\sin x \cos^2 x - 3\sin^3 x$

$= 3\sin x(2\cos^2 x - \sin^2 x)$



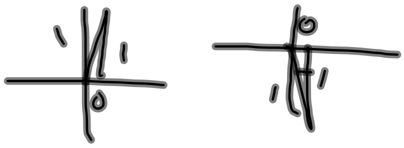


$y' = 0$

$3\sin^2 x \cos x = 0$

$\sin^2 x = 0$ $\cos x = 0$

$\sin x = 0$ $x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$



$x = 0, \pi, 2\pi, \dots$ $x \in \{ \frac{(2n+1)}{2}\pi \mid n \in \mathbb{Z} \}$

$x \in \{ n\pi \mid n \in \mathbb{Z} \}$

$y'' = 0$

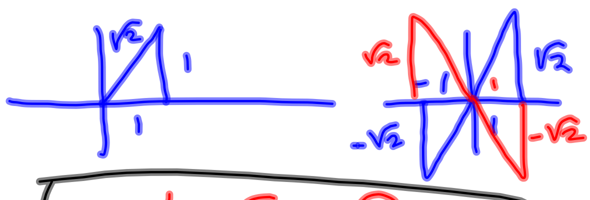
$3\sin x = 0$ or $2\cos^2 x - \sin^2 x = 0$

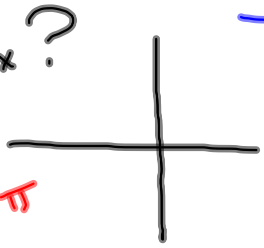
$x \in \{ n\pi \mid n \in \mathbb{Z} \}$ $(\sqrt{2}\cos x - \sin x)(\sqrt{2}\cos x + \sin x) = 0$

$\sqrt{2}\cos x = \sin x$

$\sqrt{2} = \tan x$

What's the range of arctan x?





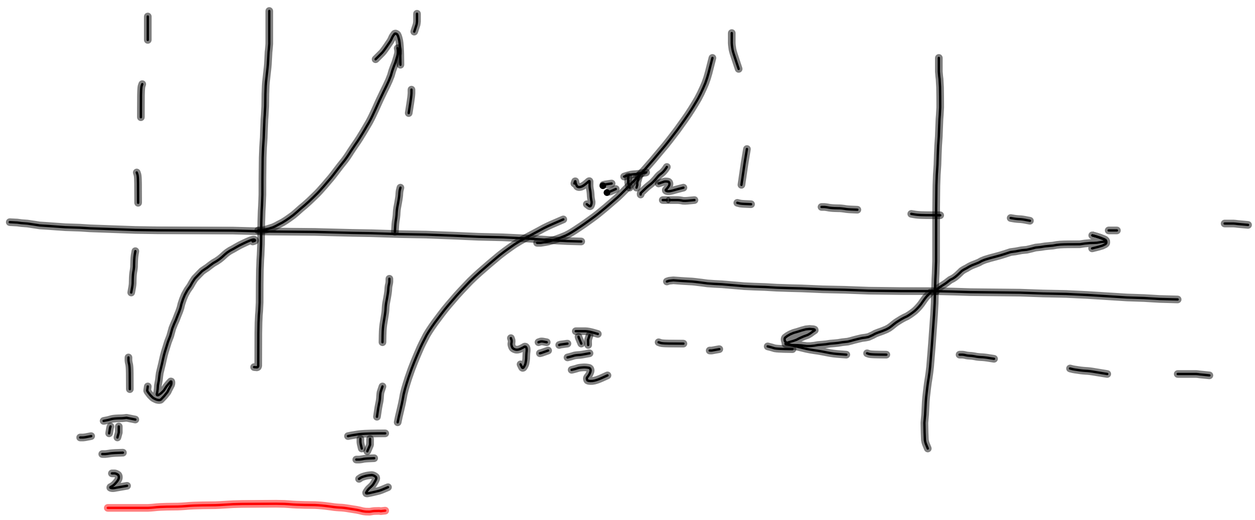
$\frac{\pi}{3} = .\overline{33}\pi$

$\arctan\sqrt{2} = ?$

$\approx .95531662$

$\approx .30408672\pi$ radians

a little under $60°$ $(\frac{\pi}{3}$ radians$)$



$\tan x$ $\tan^{-1}(x) = \arctan(x)$

$\mathcal{D} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \mathcal{R}(\tan^{-1}(x))$

$\mathcal{R} = \mathbb{R} = \mathcal{D}(\tan^{-1}(x))$

for purposes of inverting restrict $\tan x$ to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f := x \rightarrow (\sin(x))^3$

$$x \rightarrow \sin(x)^3$$

$fp := D(f)$

$$x \rightarrow 3\sin(x)^2\cos(x)$$

$solve(fp(x) = 0)$

$$0,\ 0,\ \frac{1}{2}\pi$$

$fpp := D(fp)$

$$x \rightarrow 6\sin(x)\cos(x)^2 - 3\sin(x)^3$$

$solve(fpp(x) = 0)$

$$0,\ \pi - \arccos\left(\frac{1}{3}\sqrt{3}\right),\ \arccos\left(\frac{1}{3}\sqrt{3}\right)$$

$evalf(\%)$

$$0.,\ 2.186276036,\ 0.9553166180$$

$\mathrm{Pi} - 0.9553166180$

$$\pi - 0.9553166180$$

$evalf(\%)$

$$2.186276036$$

§3.7 #s 3, 5, 7, 9, 15, 19, 35, 39

§3.6 #s 11

§3.7 #2 two #s. Diff. is 100 find min. product.

$x - y = 100 \Rightarrow y = x - 100$

Minimize $xy = x(x-100)$

$= x^2 - 100x$

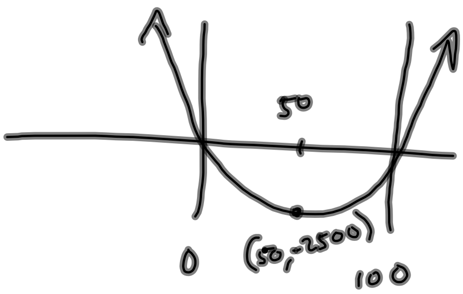
$\Rightarrow y' = 2x - 100$

$x = 50 \Rightarrow y = 50 - 100$

$= -50$

$50 - (-50) = 100$

$(50)(-50) = -2500$



#6 Minimize vertical distance between

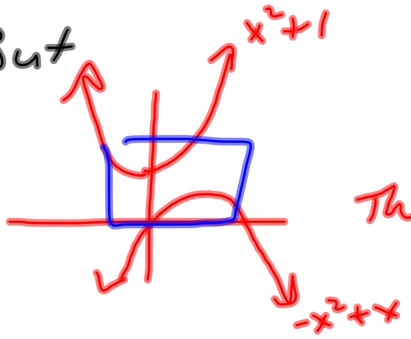
$y = x^2+1$ and $y = -x^2+x = -x(x-1)$

$$x^2+1-(-x^2+x) = 2x^2-x+1$$
$$= (2x+1)(x-1) \overset{\text{SET}}{=} 0$$
$$\Longrightarrow x = -\tfrac{1}{2},\ 1$$

New p.

The difference of the two functions.

But



They never touch!

Try again.

Vertical Distance between x^2+1 & $-x^2+x$:

$x^2+1-(-x^2+x) = 2x^2-x+1 = y$

$b^2-4ac = (-1)^2-4(2)(1) = 1-8 = -7 < 0$

$2x^2-x+1=0$ has no real solutions

So x^2+1 is ALWAYS on top.

$y' = 4x-1 \overset{\text{SET}}{=} 0$

$\Rightarrow \boxed{x=\frac{1}{4}}$ is where the vertical distance is minimized

Min. Distance is $(2(1/4)^2-\frac{1}{4}+1)$

$= \frac{2}{16}-\frac{1}{4}+1 = \frac{1-2+8}{8} = \boxed{\frac{7}{8} = \text{min dist.}}$

$g := x \to x^2 + 1$

$$x \to x^2 + 1 \qquad (9$$

$h := x \to -x^2 + x$

$$x \to -x^2 + x \qquad (10$$

$plot([h(x), g(x)], x = -3..3, color = [black, blue], thickness = [2, 2], scaling = constrained)$