Pg 19 Increasing / Decreasing : They include the endpoints, e.g.,

$f(x) = x^2$

INC : $[0, \infty)$

DEC : $(-\infty, 0]$

But NOW, in chapter 3, they would say

$$\boxed{\begin{array}{l} \text{INC: } (0, \infty) \\ \text{DEC: } (-\infty, 0) \end{array}}$$

$f' = 0$ @ the min.

§3.3 #9 $f(x) = 2x^3 + 3x^2 - 36x$

(a) inc/dec.

(b) Local Extrema

(c) Concavity & I.P.s.

(a) $f'(x) = 6x^2 + 6x - 36 \overset{SET}{=} 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x \in \{-3, 2\}$

Sign Pattern for f'

f': $+$ ↗ -3 ($=0$) $-$ ↘ 2 ($=0$) $+$ ↗

$\boxed{\text{(a) INC: } (-\infty, -3) \cup (2, \infty) \quad \text{DEC: } (-3, 2)}$

$f(2)$:

2 |	2	3	−36	0
		4	14	−36
	2	7	−18	−36 = f(2)

−3 |	2	3	−36	0
		−6	9	81
	2	−3	−27	81

$\boxed{\text{(b) } (2, -36) \text{ MIN} \quad (-3, 81) \text{ MAX}}$

(c) $f''(x) = 12x + 6 \overset{SET}{=} 0 \Rightarrow x = -\frac{1}{2}$

f'': $-$ ∩ $-\frac{1}{2}$ $+$ ∪

c. up: $(-\frac{1}{2}, \infty)$

c. down: $(-\infty, -\frac{1}{2})$

IP: $x = -\frac{1}{2}$

$-\frac{1}{2}$ |	2	3	−36	0
		−1	−1	37/2
	2	2	−37	$\frac{37}{2}$

$\boxed{(-\frac{1}{2}, \frac{37}{2}) \text{ I.P.}}$

(2, −36) MIN

(−3, 81) MAX

$\boxed{\left(-\frac{1}{2}, \frac{37}{2}\right) \text{ I.P.}}$

2^{nd} Derivative Test: Using concavity
when $f''(x)=0$ and f is smooth.

$f(x) = 2x^3 + 3x^2 - 36x$

$f'(x) = 6x^2 + 6x - 36$

$f''(x) = 12x + 6$

1^{st} derivative gave us $x = -3, 2$

2^{nd} .. says:

$f''(-3) = 12(-3) + 6 = -36 + 6 = -30 \underline{<0}$

$\underbrace{f'' < 0 \ \& \ f' = 0}_{\text{MAX.}}$

Max

$f''(2) = 12(2) + 6 = 30 > 0$

MIN

(2, -36) MIN
(-3, 81) MAX

$\boxed{\left(-\frac{1}{2}, \frac{37}{2}\right) \text{ I.P.}}$

$f(x) = 2x^3 + 3x^2 - 36x$

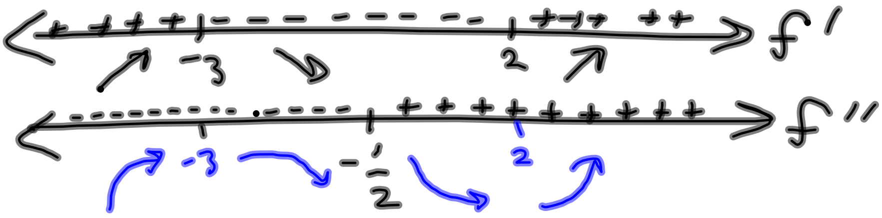
$f'(x) = 6x^2 + 6x - 36$

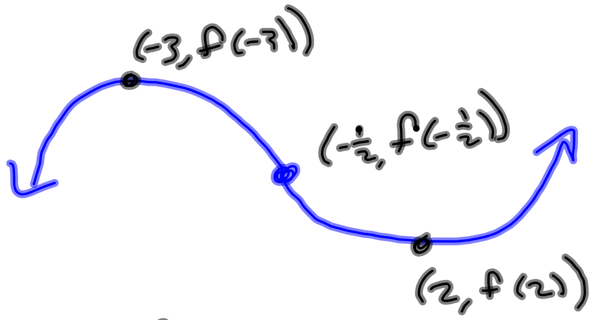
$f''(x) = 12x + 6$

Typical Test Question: Graph $f(x) = 2x^3 + 3x^2 - 36x$, showing all extremes, IP's, etc.

$f'(x) = 0 \ldots \quad x = 2, -3$

$f''(x) = 0 \ldots \quad x = -\frac{1}{2}$





Don't supply f(2) unless you get TIME on the test.

I've been using "BIG PICTURE" Analysis to get these sign patterns.
Don't be afraid to employ test values.
(Trig funcs & weird root functions)

§3.4 Limits @ Infinity

$$\lim_{x\to\infty} \frac{x^5-7x^4}{x^9+1} = 0$$

This is a <u>PROPER</u> Rational function.
$9 > 5$

$$\lim_{x\to\infty} \frac{5x^3+7x^2}{2+3x-12x^3} = -\frac{5}{12}$$

Precise Defins Pg 231. Save 'til you declare as a math major

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+5x-7}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2(1+5/x-7/x^2)}}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2}\sqrt{1+\frac{5}{x}-\frac{7}{x^2}}} = \lim_{x\to\infty} \frac{x}{|x|\sqrt{1+\frac{5}{x}-\frac{7}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{x}{x\sqrt{1+5/x-\frac{7}{x^2}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{5}{x}-\frac{7}{x^2}}} = \frac{1}{\sqrt{1}} = 1$$

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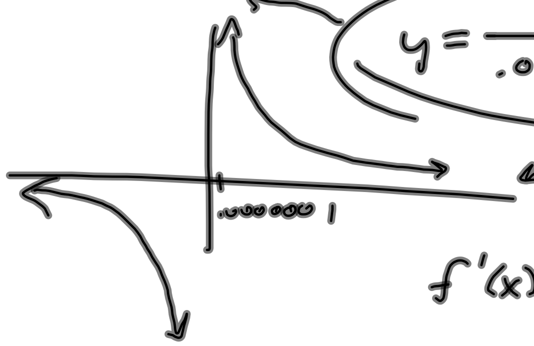
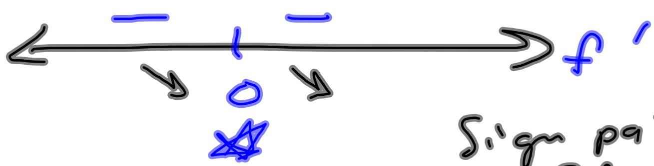
$$f(x) = \frac{1}{x}$$

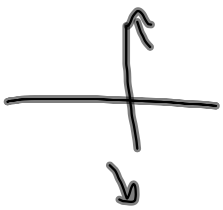
$$y = \frac{1}{.0000000001}$$



$$y = \frac{1}{10000000}$$

$$f'(x) = -\frac{1}{x^2}$$





Sign pattern plus Blowup @ $x=0$ gives you the part close to $x=0$.