Due Tues. §3.3 I #51, (Read 3,5), 8, $\underbrace{9-17, 21-25}_{\text{ODDS}}$

Due Wed. §3.3 II #s 29 - 39

Recall 1st derivative test.

$x=2$ is c.v. $\left(\begin{matrix} 2 \in \mathcal{D}(f) \\ f'=0 \text{ or } f' \nexists \end{matrix} \right)$

$f(x) = \frac{1}{x-2}$ does not have a c.v. @ $x=2$,

even though $f'(x) = -\frac{1}{(x-2)^2} \nexists$ @ $x=2$.

Example where $f(2) \exists$ but $f'(2)$ does not:

$f(x) = (x-2)^{1/2}$ or $(x-2)^{1/3}$ or

($x=2$ is an endpt. of domain in 1st, so "local extreme" doesn't apply.

$f(x) = (x-2)^{\frac{1}{3}} \Rightarrow f'(x) = \frac{1}{3}(x-2)^{-\frac{2}{3}} = \frac{1}{3(x-2)^{2/3}}$

which $\nexists$ at $x=2$

$x=2$ is c.v.

Power between 0 & 1

The Role of f''

Concave up. Positive Concavity

$f'' > 0$

"Positive Attitude"

Concave Down, Negative Concavity

$f'' < 0$

"Negative Attitude

Inflection Point. $f(c)\ \exists$

$$f''(c)=0$$

OR

$$f''(c)\ \nexists$$

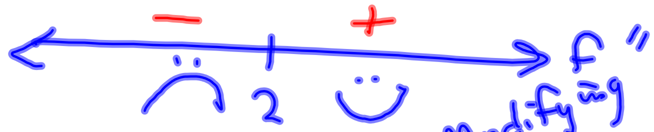
$$f(x)=(x-2)^3$$

$$f'(x)=3(x-2)^2$$

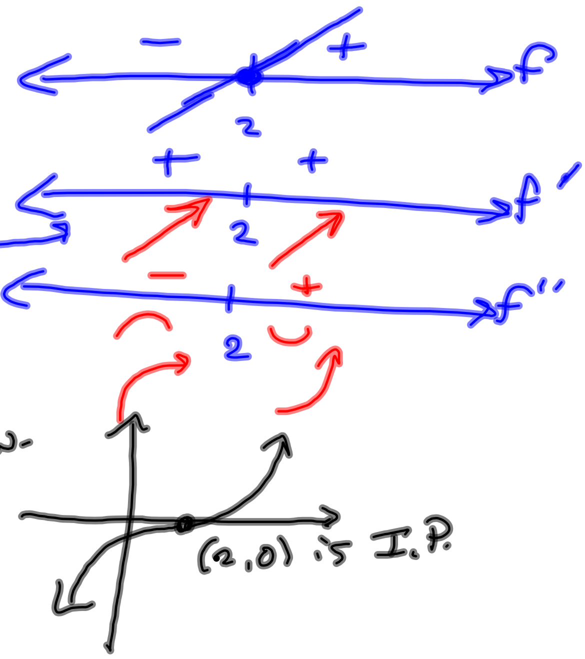
$$f''(x)=\boxed{6(x-2)^1}$$

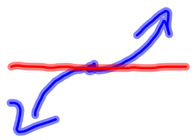
$$f''(x)=0 \implies$$

$$x=2$$



Modifying texture





For the graphing zeros of f(x) aren't as crucial right now. Shape & relative position matter most.

Sketch the graph of $f(x)= x^{\frac{2}{3}}(x-6)^{\frac{1}{3}}$

$\mathcal{D}=\mathbb{R}$

$f'(x)= \frac{2}{3}x^{-\frac{1}{3}}(x-6)^{\frac{1}{3}} + \frac{1}{3}x^{\frac{2}{3}}(x-6)^{-\frac{2}{3}}$

$= \frac{2(x-6)^{\frac{1}{3}}}{3x^{\frac{1}{3}}} + \frac{x^{\frac{2}{3}}}{3(x-6)^{2/3}}$ $\qquad LCD = x^{\frac{1}{3}}(x-6)^{\frac{2}{3}}(3)$

$= \frac{2(x-6)^{\frac{1}{3}}}{3x^{\frac{1}{3}}}\cdot\frac{(x-6)^{\frac{2}{3}}}{(x-6)^{\frac{2}{3}}} + \frac{x^{\frac{2}{3}}}{3(x-6)^{2/3}}\cdot\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

$= \frac{2(x-6)^{\frac{1}{3}+\frac{2}{3}} + x^{\frac{2}{3}+\frac{1}{3}}}{LCD}$

$= \frac{2(x-6)^1 + x^1}{LCD} = \frac{2x-12+x}{LCD} = \frac{3x-12}{LCD}$

$= \frac{3(x-4)}{3x^{\frac{1}{3}}(x-6)^{\frac{2}{3}}} = \frac{x-4}{x^{1/3}(x-6)^{2/3}}$

c.v.'s

$x-4=0$

$\boxed{x=4}$

$f'=0$

$x^{\frac{1}{3}}(x-6)^{\frac{2}{3}}=0$

$x=0,6$

$f' \nexists$

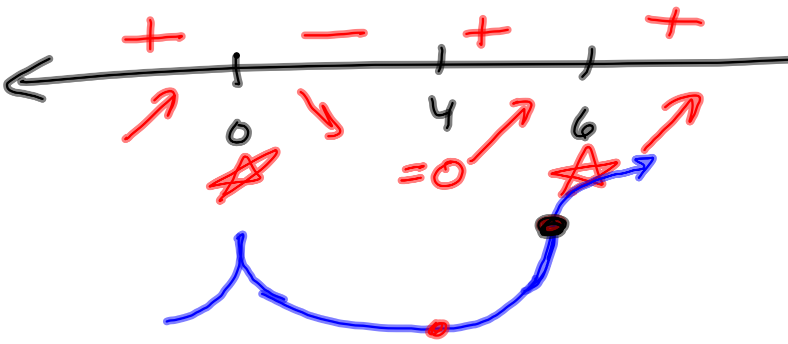
$\boxed{\text{cv's: } x=0,4,6}$

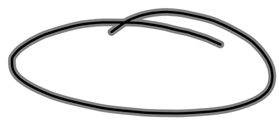
$$f'(x) = \frac{(x-4)^1}{x^{1/3}(x-6)^{2/3}}$$

ODD. changes sign

even. No sign change.

What can we say about f(x)'s graph?





$$f''(x) = \frac{1\left(x^{\frac{1}{3}}(x-6)^{1/3}\right) - (x-4)\left(\frac{1}{3}x^{-\frac{2}{3}}(x-6)^{\frac{2}{3}} + x^{\frac{1}{3}}\left(\frac{2}{3}(x-6)^{-\frac{1}{3}}\right)\right)}{\left(x^{1/3}(x-6)^{2/3}\right)^2}$$

$$= \frac{x^{\frac{1}{3}}(x-6)^{\frac{2}{3}}\left[1\,(x-6)^{-\frac{1}{3}} - \frac{(x-4)}{3}\frac{x^{-2/3}}{x^{1/3}} + \frac{2}{3}\frac{(x-6)^{-\frac{1}{3}}}{(x-6)^{2/3}}\right]}{x^{2/3}(x-6)^{4/3}}$$

owie!

$$= \frac{(x-6)^{-\frac{1}{3}} - \frac{x-4}{3}\cdot\frac{1}{x} - \frac{2(x-4)}{3(x-6)}}{x^{\frac{1}{3}}(x-6)^{\frac{2}{3}}}$$

Too painful. Hulk tired.