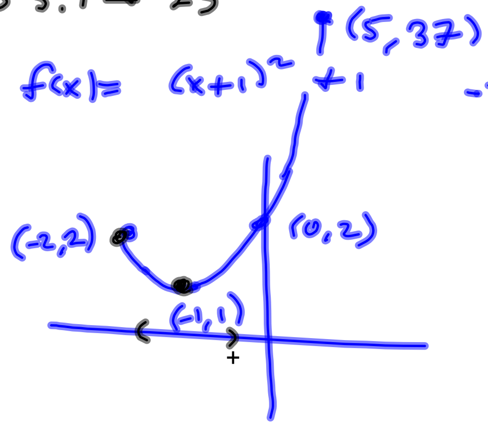
§3.1 #23

$f(x) = (x+1)^2 + 1 \quad -2 \le x < 5$



The question is
"Is (-2,2) a local max point on the graph?"
Not according to our book.

Recall: Critical Values c

$c \in \mathcal{D}(f)$ *,

$f'(c) = 0$

OR

$f'(c) \nexists$

$f(x) = \frac{x^2-1}{x+5}$ $\mathcal{D}(f) = \{x \mid x \neq -5\}$

$\Longrightarrow f'(x) = \frac{2x(x+5) - (x^2-1)(1)}{(x+5)^2}$

$= \frac{2x^2+10x-x^2+1}{(x+5)^2}$

$= \frac{x^2+10x+1}{(x+5)^2}$

$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4}\sqrt{6}$
$= 2\sqrt{6}$

$f'(x) = 0$: $x^2+10x+1 = 0$

$x^2+10x+5^2 = -1+25$

$(x+5)^2 = 24$

$x+5 = \pm\sqrt{24} = \pm 2\sqrt{6}$

$x = -5 \pm 2\sqrt{6}$

$f'(x) \nexists$: $(x+5)^2 = 0$

$x+5 = 0$

$x = -5$.

Critical Values are $-5 \pm 2\sqrt{6}$.

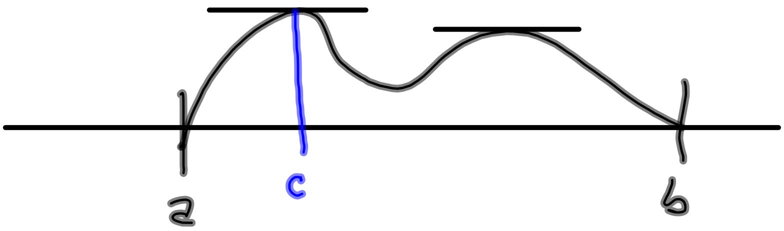
$x=-5$ isn't in Domain of f.

§3.2 MVT

Rolle's Theorem

f cnt^{s} on $[a,b]$

f difb^{l} on (a,b)



$\longrightarrow \exists\, c \in (a,b) \ni f'(c) = 0.$

This is ONLY an assertion of existence.

No recipe for finding c.

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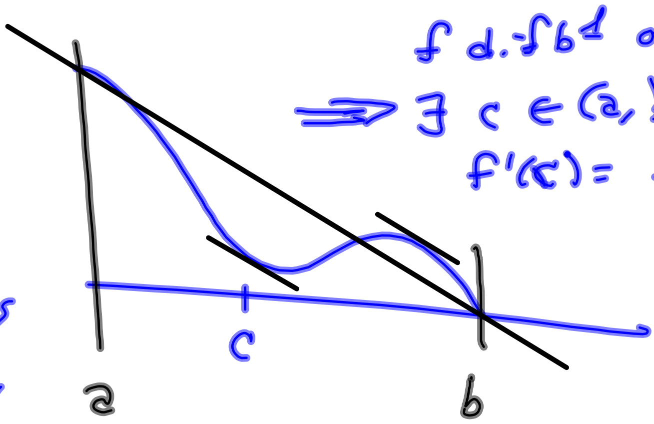
Mean Value Theorem: f cnts on $[a,b]$

f diffble on (a,b)

$\Longrightarrow \exists\, c \in (a,b) \ni$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



No recipe for c, we just know it's there, somewhere.

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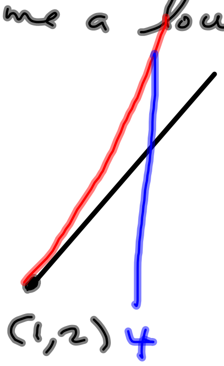
The racetrack principle:

If I'm ahead and I run faster, I stay ahead.

Suppose $f(1)=2$ and $f'(x) \geq 3$

Give me a lower bound on $f(4)$.



$f'(x) \geq 3$

$y=3(x-1)+2$ is a line that $f(x)$ stays above!

$y=3(4-1)+2$

$=3(3)+2$

$=11$ is a floor underneath whatever $f(4)$ is.

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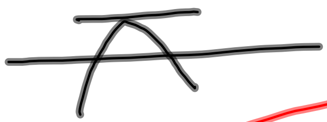
#18 Show that

$2x-1-\sin x = 0$ has exactly 1 root.

$f(x) = 2x-1-\sin x$

At MOST one crossing. $\leftarrow$ $f'(x) = 2-\sin x > 0$ everywhere,



AND $f(4) = 2(4)-1-\sin(4)$

$= 7-\sin(4) > 0$

$f(-2) = 2(-2)-1-\sin(-2)$

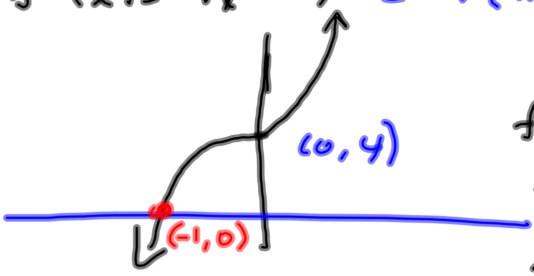
$= -5-\sin(-2) < 0$

So there's at Least one.

so exactly one crossing of the x-axis.

#20 Show that x^4+4x+C has at <u>MOST</u> two real roots.

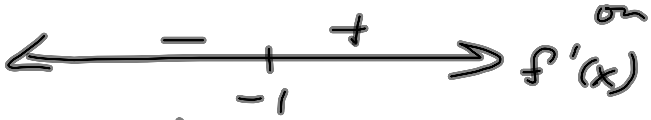
$f'(x) = 4x^3+4 = 4(x^3+1) = 4(x+1)\underbrace{(x^2-x+1)}_{\text{No real roots.}}$

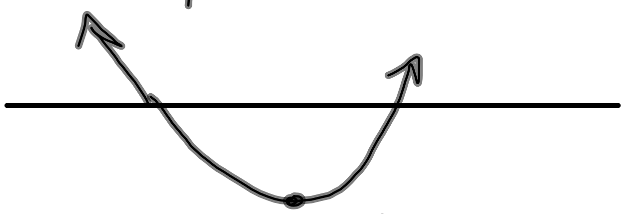


Quick Sketch of $f'(x)$

$f(x) = x^4+4x+C$

$f'(x)$ is always increasing, but more relevant, it has only ONE x-intercept, so only ONE max/min on the graph of $f(x)$.





$(-1, f(-1))$ is a min.

§3.2 #s 1–5, 9–11, 15, 17, 19, 25, 27