Test 1 #5e

$y = \cot(\sec(x^2-5)) \implies$

$y' = -\csc^2(\sec(x^2-5))(\sec(x^2-5)\tan(x^2-5))(2x)$

$\frac{d}{dx}\left[f(g(h(x)))\right] = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dx}$

3 $\lim_{x \to 5} (2x-7) = 3$

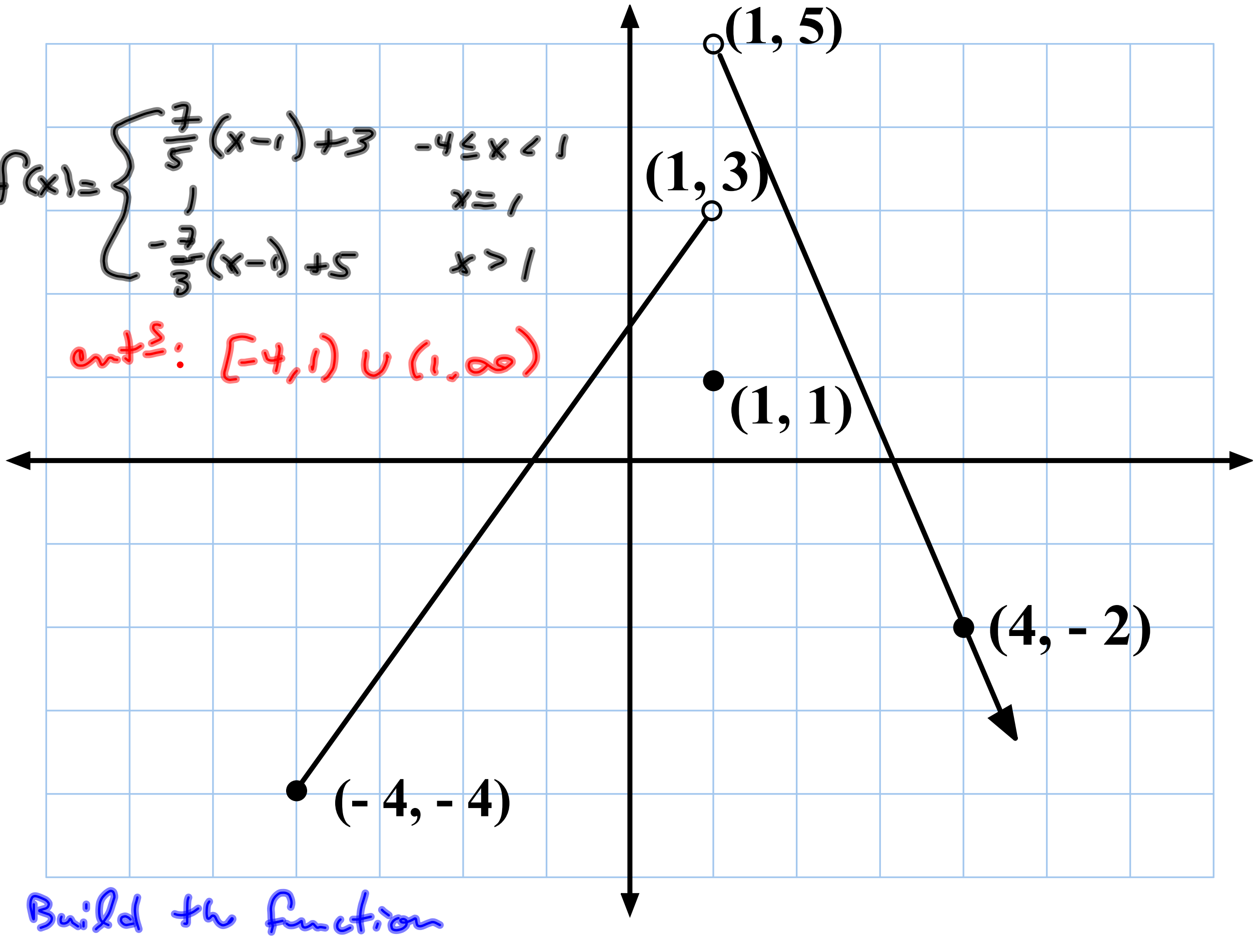
Proof Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$

Then any time $0 < |x-5| < \delta$, we have

$|(2x-7)-3| = |2x-10| = 2|x-5| < 2\delta = \varepsilon$ ∎

$$f(x)=\begin{cases}\frac{7}{5}(x-1)+3 & -4\le x<1\\ 1 & x=1\\ -\frac{7}{3}(x-1)+5 & x>1\end{cases}$$

ans: $[-4,1) \cup (1,\infty)$



Build the function

$-4 \le x < 1$

$m = \frac{-4-3}{-4-1} = \frac{-7}{-5} = \frac{7}{5}$

$y = \frac{7}{5}(x-1)+3$

$x=1, y=1$

$1 < x < \infty$

$m = \frac{-2-5}{4-1} = \frac{-7}{3}$

$y = -\frac{7}{3}(x-1)+5$

§3.1 Optimization

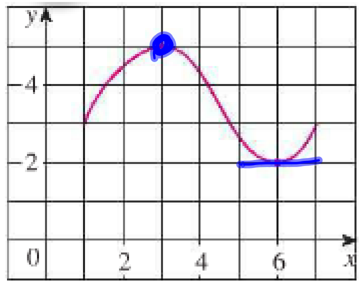
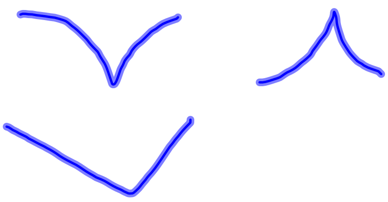


FIGURE 1



① **Absolute max** on $\mathcal{D}$ at $x=c$ means $f(c) \geq f(x)$ $\forall x \in \mathcal{D}$

(for every x in the Domain)

Abs. Min: $f(c) \leq f(x)$ $\forall x \in \mathcal{D}$.

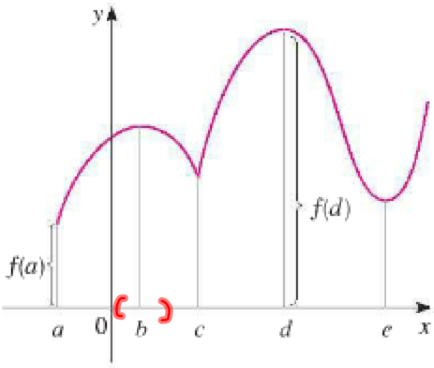


FIGURE 2
Abs min $f(a)$, abs max $f(d)$,
loc min $f(c)$, $f(e)$, loc max $f(b)$, $f(d)$

② **Local max** at c means $f(c) \geq f(x)$ $\forall x$ on an open interval containing c.

Likewise local min, $f(c) \leq f(x)$ $\forall x \in (a,b)$

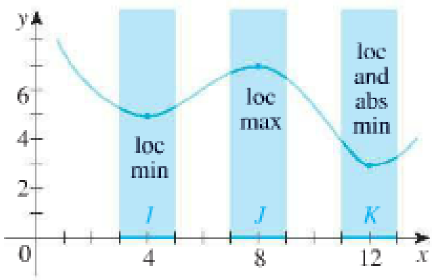


FIGURE 3

③ Extreme Value Theorem

Every continuous function on a closed interval has an abs. min and an abs. max on the interval.

Fermat's Theorem - If $f(c)$ is a max/min and $f'(c)$ $\exists$, then $f'(c)=0$.

(Sometimes you can have a max where $f'(c)$ $\nexists$)

Critical #s: Any x where $f'(x)=0$ OR $f'(x)$ $\nexists$

Optimization Recipe on closed interval $[a,b]$

1. Critical #s, c. Find the $f(c)$'s
2. Find $f(a)$ & $f(b)$
3. Biggest is max.

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#s 45-56

Find Abs. Max & Min

46 $f(x) = -2x^3 + 54x + 5$ on $[0, 4]$

$f'(x) = -6x^2 + 54 \overset{\text{SET}}{=} 0$

$6x^2 - 54 = 0$

$x^2 - 9 = 0$

$x = \pm 3$

$x = 0$

0 |	-2	54	5
		0	0
	-2	54	$\boxed{5 = f(0)}$

$x = 4$:

4 |	-2	54	5
		-8	184
	-2	46	$\boxed{189 = f(4)}$

$f(3) = -2(3)^3 + 54(3) + 5$

$= -54 + 162 + 5$

$= \boxed{221 = f(3)}$ ABS MAX

-3 |	-2	54	5
		6	-180
	-2	60	$\boxed{-175 = f(-3)}$

ABS MIN

$\begin{array}{r} {}^{2}46 \\ \times\ 4 \\ \hline 184 \end{array}$

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§3.1 #s 5, 6, 7, 9, 15–31, 35–41, 47, 53

DUE THURSDAY

Find Max of $f(x) = \frac{1}{x}$ on $[-1, 1]$

There isn't one!

But what about EVT?

Doesn't satisfy continuity requirement on $[-1, 1]$.

#42 Find c.v.s

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$y = \pm\sqrt{1-x^2}$$

$$g(x) = \sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}}$$

$$y' = \tfrac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \frac{-x}{\sqrt{1-x^2}}$$

$y' = 0$: $-x = 0 \Longrightarrow \boxed{x = 0}$

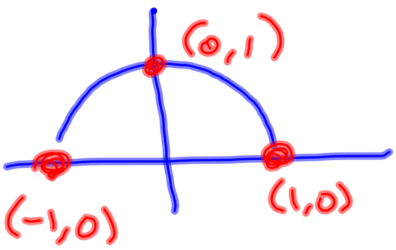
$y' \nexists$: $\sqrt{1-x^2} = 0$

$$1 - x^2 = 0$$

$$x^2 = 1$$

$$\boxed{x = \pm 1}$$

All that was asked



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(55) $f(t) = 2\cos t + \sin(2t)$ on $[0, \frac{\pi}{2}]$

$f(0) = 2$

$f(\frac{\pi}{2}) = 0$ → MIN

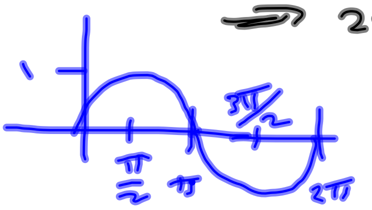
$f'(t) = -2\sin t + 2\cos(2t)$ $\exists\ \forall x$, so only looking for $f'(t) = 0$

$\overset{\text{set}}{=} 0 \Longrightarrow 2\cos(2t) - 2\sin(t) = 0$

$\Longrightarrow \cos(2t) - \sin(t) = 0$

$\Longrightarrow 1 - 2\sin^2(t) - \sin t = 0$

$\Longrightarrow 2\sin^2 t + \sin t - 1 = 0$



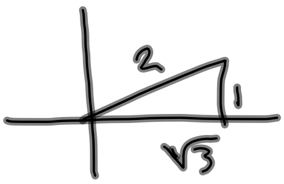
$2u^2 + u - 1 = 0$

$(2u - 1)(u + 1) = 0$

$\Longrightarrow 2\sin t = 1$ $\sin t = -1$ Never, on $[0, \frac{\pi}{2}]$

$\sin t = \frac{1}{2}$

$t = \frac{\pi}{6}$



$f(\frac{\pi}{6}) = 2\cos(\frac{\pi}{6}) + \sin(2 \cdot \frac{\pi}{6})$

$= 2 \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \boxed{\frac{3\sqrt{3}}{2} = f(\frac{\pi}{6})}$ → MAX