2.5 #21

$$y=\left(\frac{x^2+1}{x^2-1}\right)^3 \Longrightarrow$$

$$y' = 3\left(\frac{x^2+1}{x^2-1}\right)^2\left(\frac{2x(x^2-1)-(x^2+1)(2x)}{(x^2-1)^2}\right) = \frac{df}{dg}\cdot\frac{dg}{dx}$$

STOP!

$$f(g)=g^3 \qquad f'(g)=3(g)^2=\frac{df}{dg}$$

$$g(x)=\frac{x^2+1}{x^2-1}$$

$$g'(x)=\frac{2x(x^2-1)-(x^2+1)(2x)}{(x^2-1)^2}=\frac{dg}{dx}$$

cleanup

$$=3\,\frac{(x^2+1)^2}{(x^2-1)^2}\left(\frac{2x^3-2x-2x^3-2x}{(x^2-1)^2}\right)$$

$$=3\,\frac{(x^2+1)^2(-4x)}{(x^2-1)^4}=\frac{-12x(x^2+1)^2}{(x^2-1)^4}$$

Doug did it in an "insane" way.

$$y=\left(\frac{x^2+1}{x^2-1}\right)^3=\frac{(x^2+1)^3}{(x^2-1)^3}=(x^2+1)^3(x^2-1)^{-3} \Longrightarrow$$

$$y'=3(x^2+1)^2(2x)(x^2-1)^{-3}+(x^2+1)^3\left[-3(x^2-1)^{-4}\right](2x)$$

$$=\frac{6x(x^2+1)^2}{(x^2-1)^3}-\frac{6x(x^2+1)^3}{(x^2-1)^4}$$

$$\frac{x^2-1}{x^2-1}\cdot\frac{6x(x^2+1)^2}{(x^2-1)^3}-\frac{6x(x^2+1)^3}{(x^2-1)^4}$$

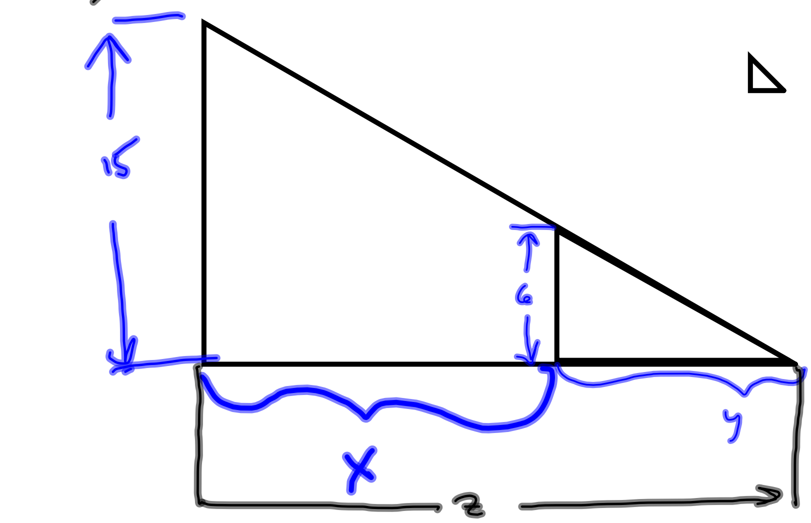
$$=\frac{6x(x^2+1)^2(x^2-1)-6x(x^2+1)^3}{(x^2-1)^4}$$

$$=\frac{6x(x^2+1)^2\left[x^2-1-(x^2+1)\right]}{(x^2-1)^4}=\frac{-12x(x^2+1)^2}{(x^2-1)^4}$$

§2.8 #13

§2.9 #7

#13



$\frac{dx}{dt} = 5\ \frac{ft}{s}$

want

$\left.\frac{dz}{dt}\right|_{x=40}$

$\frac{15}{x+y} = \frac{6}{y}$

$15y = 6x + 6y$

$15y' = 6x' + 6y'$

$9y' = 6x'$

$y' = \frac{6}{9}x' = \frac{2}{3}x'$

$z = x + y$

$z' = (x+y)'$

$= x' + y'$

so $\frac{dz}{dt} = x' + y'$

$= x' + \frac{2}{3}x'$

$= \frac{5}{3}x'$

$= \frac{5}{3} \cdot 5 = \boxed{\frac{25}{3}\ ft/s}$

§2.9 #7

For what values of x will the Linear approximation be w/in .1 of actual, for $\sqrt[4]{1+2x} = (2x+1)^{\frac{1}{4}}$

Given $L_0(x) = \frac{1}{2}x+1$

we're in the neighborhood of $x=0$.

$$|L_0(x) - f(x)| \overset{\text{want}}{<} .1$$

$$\left|\tfrac{1}{2}x+1-(2x+1)^{\frac{1}{4}}\right| < .1$$

This is a technology question.

$$\underset{y=-.1}{-.1} < \underbrace{\tfrac{1}{2}x+1-\sqrt[4]{2x+1}}_{\substack{\text{Graph} \\ y=\text{this}}} < \underset{y=.1}{.1}$$

here's a CAS result:

$f := x \rightarrow \left|\frac{1}{2}\cdot x + 1 - \sqrt[4]{2\cdot x+1}\right|$

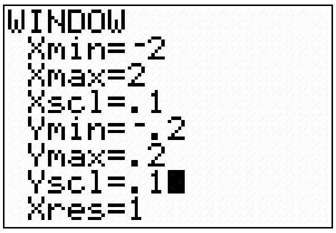
$x \rightarrow \left|\frac{1}{2}\,x + 1 - (2\,x+1)^{1/4}\right|$

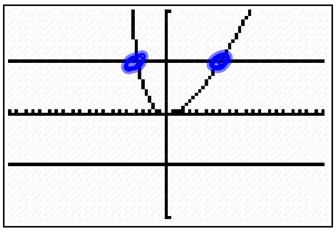
$solve(f(x) = 0.1, x)$

$-0.368934652,\ 0.677669220,\ -0.156673168 + 0.5099027290\,I,\ -0.156673168 - 0.5099027290\,I$

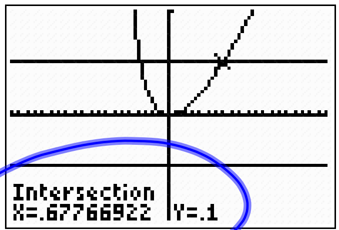
Not Real

$x \in (-.368934652, 0.677669220)$





Find the x-coords.



Says x=.67766922, which is Maple answer.

Still need the left endpoint.

Test Review in earnest.

Limit Questions:

$$\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$$

$$\lim_{x\to 2} \frac{x^2-3x-7}{x-2}$$

Numerical Limit. Plug in, e.g., $x = 2.001$ to see...

ε-δ Proof

Prove $\lim_{x\to 3}(5x-7) = 8$

Bonus Prove $\lim_{x\to 3}(x^2+5x) = 24$

Derivative by the definition

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

for $f(x) = ax^2+bx+c$, $\frac{1}{x}$, $\sqrt{x}$

$$\frac{d}{dx}\left[x^n,\ f(x)^n,\ \frac{f(x)}{g(x)},\ \text{trig}(x),\ f(x)g(x),\ f(g(x))\right]$$

Find $\frac{dy}{dx}$

$$y = \tan^4\left(\frac{\sin x + x^3 - 5x}{\sec x}\right) \Rightarrow y' =$$

$$4\left(\tan\left(\frac{\sin x + x^3 - 5x}{\sec x}\right)\right)^3\left(\frac{(\cos x + 3x^2 - 5)(\sec x) - (\sin x + x^3 - 5x)(\sec x \tan x)}{\sec^2 x}\right)$$

What's wrong with this?

Missed $\sec^2\left(\frac{\sin x + x^3 - 5x}{\sec x}\right)$ factor.