§2.9 #5 $L_0(x) =$

$f(x) = \sqrt{1-x}$

~~Approx: $f(.9)$, $f(.95) = ?$~~
~~$L_0(.9)$, $L_0(.95)$~~

$L_a(x) = f'(a)(x-a) + f(a)$

$f(x) = (1-x)^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(1-x)^{-1/2}(-1)$

$f(0) = \sqrt{1} = 1$

$f'(0) = \frac{-1}{2\sqrt{1-0}} = -\frac{1}{2} = m = f'(a)$

$L_0(x) = -\frac{1}{2}(x-0) + 1$

Use this to approximate $\sqrt{.9}$ & $\sqrt{.99}$

$.9 = 1 - .1$ so $x = .1$

$.99 = 1 - .01$ so $x = .01$

$L_0(x) = -\frac{1}{2}x + 1$

$L_0(.1) = -\frac{1}{2}(.1) + 1 = .95$

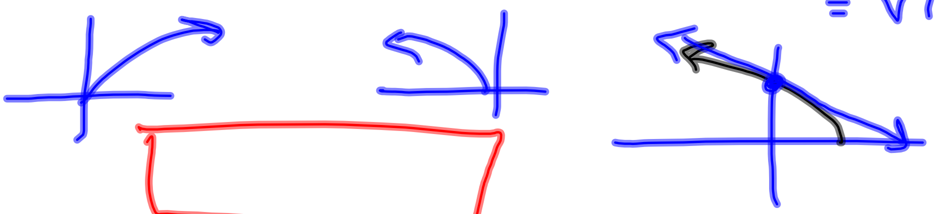
$L_0(.01) = -\frac{1}{2}(.01) + 1 = .995$

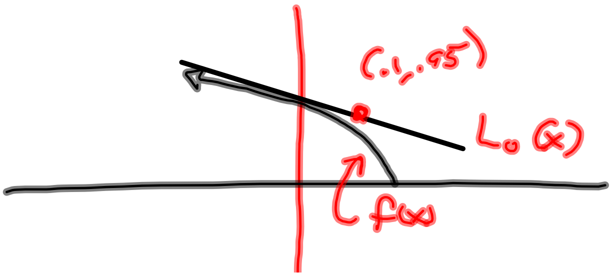
$$\begin{array}{r} 1.000 \\ -.005 \\ \hline .995 \end{array}$$

$1 - x = -x + 1$
$= -(x-1)$

$\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \sqrt{-(x-1)}$

$= \sqrt{1-x}$





$\sqrt{.9}$

0.9486832981

$\sqrt{.99}$

0.9949874371

$\sqrt{1-x}$ is odd choice for approximating $\sqrt{.9}$, $\sqrt{.99}$. Why not just $f(x)=\sqrt{x}$ and $a=1$, instead of $f(x)=\sqrt{1-x}$ and $a=0$?

Tan Line to $f(x)=\sqrt{x}$ @ $x=1$:

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad$ $f(x) = x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(1) = \frac{1}{2} = f'(a) = m$

$f(1) = 1 \qquad (x_1, y_1) = (1,1) = (a, f(a))$

$y = m(x - x_1) + y_1$ $\qquad$ Easier for ME to see.

$L_1(x) = f'(1)(x-1) + 1$

$= \frac{1}{2}(x-1) + 1$

$L_1(.99) = \frac{1}{2}(.99-1) + 1 = \frac{1}{2}(-.01) + 1$

$= -.005 + 1$

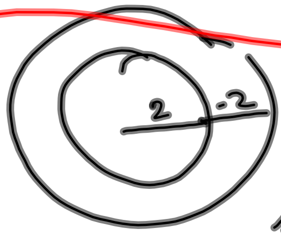
$= .995$

Use of Linear Approximation & Differentials.

The radius of a tree is measured to be 2 ft, with a possible error $\pm .2$ ft. Approximate the error in calculated cross-sectional area.

$$A = \pi r^2$$

$\boxed{dA = 2\pi r\, dr}$ is the differential of area.

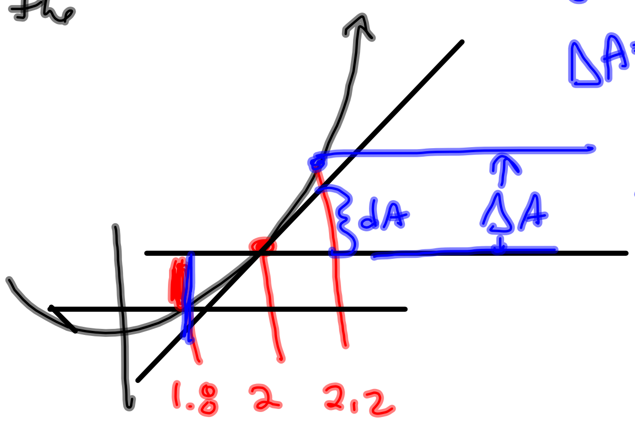


$$\Delta A \approx dA = 2\pi(2)(\pm .2)$$
$$= .8\pi \text{ ft}^2$$
$$\approx 2.512 \text{ ft}^2$$

$$\frac{dA}{dr} = 2\pi r \Longrightarrow dA = 2\pi r\, dr$$

$$\begin{array}{r} 3.14 \\ 8 \\ \hline 25.12 \end{array}$$

Here's the pic!

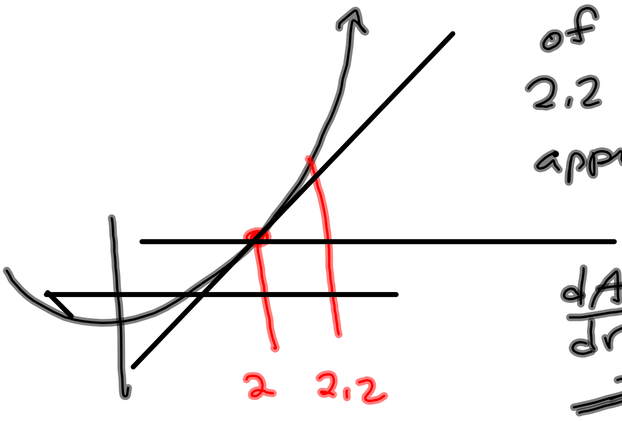


Estimating $\Delta A = A(2.2) - A(2)$

or

$\Delta A = A(1.8) - A(2)$

with $dA = 2\pi(2)(.2)$



Approximate the area of a disk of radius 2.2 ft, by the Linear approximation to

$A = \pi r^2$ at $r = 2$.

$\frac{dA}{dr} = 2\pi r = A'(r)$

$\Longrightarrow A'(2) = 2\pi(2)$
$= 4\pi = m$

$A(2) = \pi(2)^2$
$= 4\pi$

$L_2(x) = 4\pi(x-2) + 4\pi$
$= 4\pi(0.2) + 4\pi$
$= .8\pi + 4\pi$
$= 4.8\pi$

$\Delta A \approx dA =$

Differential $= A(2.2) - A(2)$ the change

Linear Approx $\approx A(2.2)$, via tangent line

$\sin(92^\circ) = \sin(x+\Delta x)$

ALWAYS USE RADIANS!

$92^\circ = 90^\circ + 2^\circ$

$= \frac{\pi}{2} + 2 \cdot \frac{\pi}{180}$ Radians.

$= \frac{\pi}{2} + \frac{\pi}{90}$

$f(x) = \sin x$, $x = \frac{\pi}{2} = a$, $\Delta x = \frac{\pi}{90}$

$f'(x) = \cos x$

$f'(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) = 0$

$f(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

$L_{\frac{\pi}{2}}(x) = 0(x - \frac{\pi}{2}) + 1$

$L_{\frac{\pi}{2}}(x) = 1$ So it thinks $\sin(92^\circ) = \sin(83^\circ) = 1$!

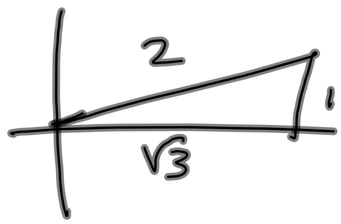
$$\sin(27^\circ) = \sin(30^\circ - 3^\circ)$$

$$x = 30^\circ = \frac{\pi}{6}$$

$$\Delta x = -3^\circ = -\frac{3^\circ \pi}{180^\circ/\text{rad}} = -\frac{\pi}{60}\ \text{rad.}$$

$$f(x) = \sin x, \quad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$f'(x) = \cos x, \quad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$



$$L_{\frac{\pi}{6}}(x) = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}$$

$$\sin 27^\circ \approx \frac{\sqrt{3}}{2}\left(\frac{\pi}{6} - \frac{\pi}{60} - \frac{\pi}{6}\right) + \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}\left(-\frac{\pi}{60}\right) + \frac{1}{2} = -\frac{\sqrt{3}\pi}{120} + \frac{1}{2}$$