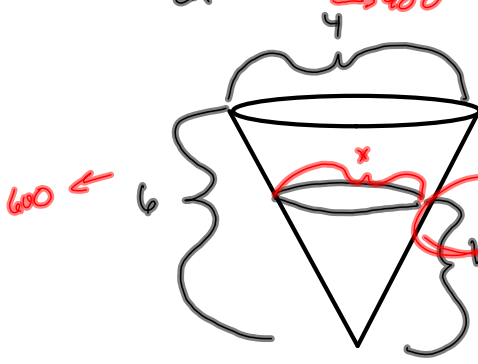
§2.8 #23

$\frac{dV}{dt} = 10000 \frac{cm^3}{min}$ → 400

leaking so $-10000 \frac{cm^3}{min}$

Pumping H_2O in @ constant rate, K.



$h=2 \Rightarrow \frac{dh}{dt} = 20 cm/min$

$\frac{6}{4} = \frac{2}{x} = \frac{3}{2} \Rightarrow x = \frac{4}{3}$

Find rate of water being pumped into the tank K

$V = \frac{1}{3}\pi r^2 h$ Each is off by a factor of 100! 100^2

$-10{,}000 \frac{cm^3}{min} + \frac{dK}{dt} = \frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

Want $\frac{dK}{dt}$ = rate the H_2O's pumping in = ?

$10000\left[\frac{2}{3}\pi \cdot \frac{2}{3} \cdot 2 \cdot \frac{dr}{dt}\right] + \frac{1}{3}\pi\left(\frac{2}{3}\right)^2(20) = ? - 10{,}000$

$\frac{4}{6} = \frac{r}{h}$

$4h = 6r$

$\frac{2}{3}h = r$

$\frac{2}{3}\frac{dh}{dt} = \frac{dr}{dt} = \frac{2}{3}(20) = \frac{40}{3}$

$10^4\left[\frac{8}{9}\pi \cdot \frac{40}{3} + \frac{80\pi}{3}\right] = ? 160 - 10000 \cdot 180$

$10^4\left[\frac{320\pi}{27} + \frac{720\pi}{27}\right] = ? -10000$

$10^4\left(\frac{1040\pi}{27}\right) + 10000 = ?$

```
10^4*1040/27
      385185.1852
Ans*π
      1210094.948
Ans+10000
      1220094.948
```

Used r like a diameter. I'm off by a factor of 2 on my radius:

$\frac{4}{6} = \frac{d}{h} = \frac{2r}{h}$

$4h = 12r$

$\frac{4}{12}h = r = \frac{1}{3}h$, not $\frac{2}{3}$!

Couldn't retro-fit this, quite, but we have all the pieces.

① cm-vs-m

② $r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$

$r = \frac{1}{8}h$, $h = 200 \Rightarrow$ $r = \frac{200}{3}$

$\frac{dr}{dt} = \frac{1}{8}\frac{dh}{dt}$

$r' = \frac{1}{8} \cdot 20\ \text{cm/min} = \frac{20}{3}$

$\frac{2}{3}\pi\left(\frac{200}{3}\right)(200)\left(\frac{20}{3}\right)$

$+ \frac{1}{3}\pi\left(\frac{200}{3}\right)^2(20)$

$= \frac{1}{3}\pi\left[\frac{2 \cdot 200^2 \cdot 20}{9} + \frac{(200^2)(20)}{3}\right] \approx 465421.1339$

off by factor of 5!

??

$= ? - 10{,}000 \Rightarrow$

$? =$ Still not getting the "right" answer.

Book Method:

Use similar triangles to eliminate $r = \frac{1}{8}h \Rightarrow$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{8}h\right)^2 h$

$= \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$

$= \frac{1}{9}\pi(200)^2(20) \approx$

$\frac{5(20)(200^2)}{9}\pi$

```
*20
          977777.7778
2*200²*20/9+200²
*20/3
          444444.4444
Ans*π/3
          465421.1339
```

This problem was instructive:

Keep track of units cm-vs-m

Look to eliminate a variable, if possible.

§2.6 #26 Like #25, sort of.

Find tan. line to $\sin(x+y) = 2x-2y$ @ (π,π)

$(\cos(x+y))(1+y') = 2-2y'$ diff wrt x

$y'\cos(x+y) + \cos(x+y) = 2-2y'$

$y'\cos(x+y) + 2y' = 2-\cos(x+y)$

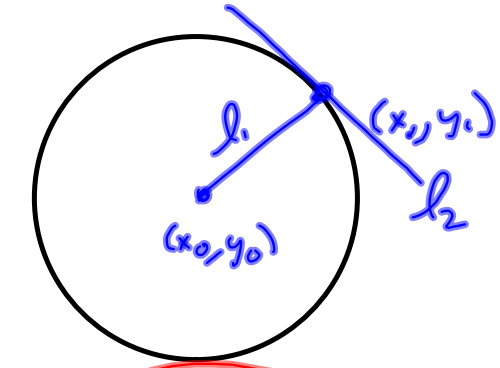
$y'(\cos(x+y)+2) = 2-\cos(x+y)$

$\Rightarrow \quad y' = \dfrac{2-\cos(x+y)}{\cos(x+y)+2}\Bigg|_{(\pi,\pi)} = \dfrac{2-\cos(2\pi)}{\cos(2\pi)+2} = \dfrac{1}{3} = m$

(π,π)

oo $\boxed{y = \frac{1}{3}(x-\pi)+\pi}$

$y = m(x-x_1)+y_1$



$x^2+y^2=r^2$

$2x+2yy'=0$

$y'=-\frac{x}{y}$

$y'\Big|_{(x_1,y_1)} = -\frac{x_1}{y_1}$

$m_1 = \frac{y_1-y_0}{x_1-x_0}$

If $x_0=y_0=0$

you see

$m_1 = -\frac{1}{m_2}$.

Sep 24-1:02 PM