§2.9 Linear Approx. & Differentials.

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

If $\Delta x = dy$, then

$$\Delta y \approx dy$$

$$\frac{f(x+\Delta x) - f(x)}{x+\Delta x - x} \approx \frac{dy}{dx}$$

$$\frac{f(x+\Delta x) - f(x)}{dx} \approx \frac{dy}{dx} = f'(x)$$

Multiply by dx

$$f(x+\Delta x) - f(x) \approx dy = \underbrace{f'(x)}_{\text{steepness}}\ \underbrace{dx}_{\text{small horizontal distance}}$$

is the differential of y.

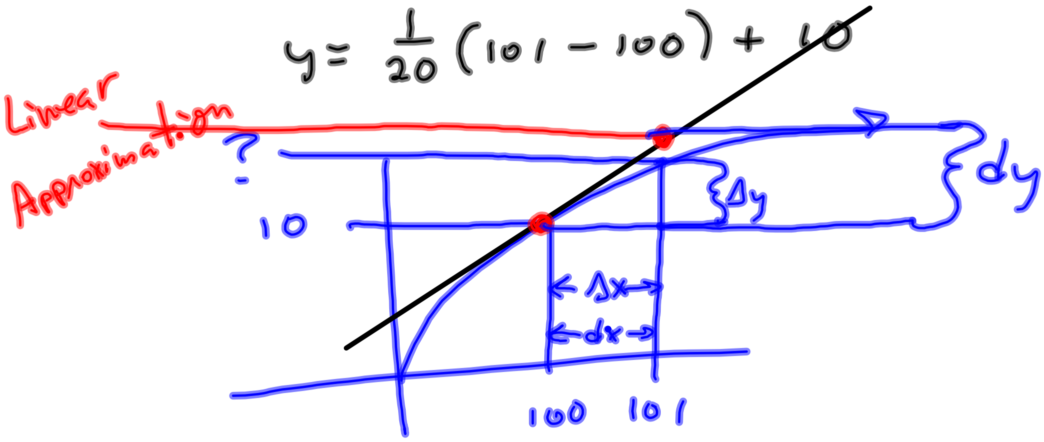
$f(x+\Delta x) \approx f'(x)\,dx + f(x)$

$\sqrt{101} \approx \frac{1}{2\sqrt{100}} \cdot 1 + \sqrt{100}$

$y = m(x - x_1) + y_1$

$y = f'(x)(101 - 100) + 10$

$y = \frac{1}{20}(101 - 100) + 10$



$\boxed{dy = f'(x)\,dx}$ The differential of y

$y = L_a(x) = f'(a)(x-a) + f(a)$ The tangent line or Linear Approximation to f(x) @ x=a.

GOOD PREDICTIONS for f(x) close to x = a

Approximate $\sqrt{103}$ using tan. line or differentials.

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$a = 100 \quad x = 103$

$\Delta x = dx = 3$

$L_{100}(x) = f'(a)(x-a) + f(a)$

$= \frac{1}{2\sqrt{100}}(103-100) + \sqrt{100}$

$= \frac{1}{20}(3) + 10 = 10 + \frac{3}{20} = \frac{203}{20} = 10.15$

Maple Says: **10.14889157**

Sep 20-12:28 PM

Same Thing using differentials

$f(x+\Delta x) - f(x) = \Delta y \approx dy = f'(x)dx$

$dy = f'(x)\,dx = \frac{1}{2\sqrt{100}} \cdot 3 = \frac{3}{20}$

$x = 100, \ dx = \Delta x = 3$

And so $f(x+\Delta x) \approx f(x) + dy$

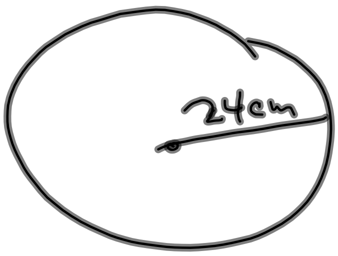
$= \sqrt{100} + \frac{3}{20}$

$= 10 + \frac{3}{20} = 10.15$

Sometimes all we want is Δy

§2.9 #32 The radius of a disk is 24 cm +/- 0.2cm.

(a) Use differentials to approximate the the error in calculated area of the disk.



$f(r) = \pi r^2$

$y = \pi r^2$

$\frac{dy}{dr} = 2\pi r$

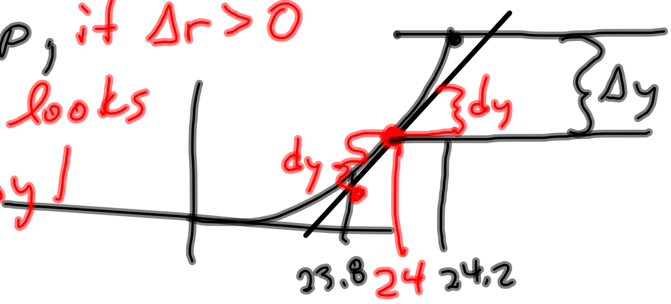
$dy = 2\pi r \, dr$

$\Delta y \approx dy = 2\pi (24)(\pm .2)$

$= \pm 9.6\pi$

is an underestimate, b/c πr^2 is concave up, if $\Delta r > 0$

If $\Delta r < 0$ it looks like our $|dy| > |\Delta y|$



Relative Error

$$= \frac{dy}{y} = \frac{\pm 9.6\pi}{\pi (24)^2} = \pm \frac{9.6}{24^2}$$

§2.9 #5 3-35 odds.