§ 2.5 #70

g is twice-difbl

$f(x) = x\,g(x^2)$. Find $f''(x)$ in terms of g, g' & g''

Chain rule on $g(x^2)$

$$\frac{d}{dx}\left[f(x)\right] = 1\,g(x^2) + x\,\boxed{g'(x^2)}\,\boxed{2x}$$

$$\frac{dg}{dx^2} \qquad \frac{dx^2}{dx}$$

Relabel:

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$\frac{d}{dx}\left[g(x^2)\right] = \frac{dg}{dx^2}\cdot\frac{dx^2}{dx}$$

So $f'(x) = g(x^2) + 2x^2 g'(x^2)$.

Now $f''(x) = g'(x^2)\cdot 2x + 4x\,g'(x^2) + 2x^2 g''(x^2)\cdot 2x$

Product + Chain Rule on

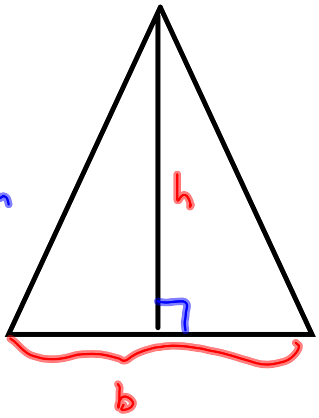
$$\frac{d}{dx}\left[2x^2 g'(x^2)\right] = \frac{d}{dx}\left[2x^2\right]g'(x^2) + 2x^2\cdot\frac{d}{dx}\left[g'(x^2)\right]$$

$$= 4x g'(x^2) + 2x^2 g''(x^2)\,2x$$

$$= \boxed{4x g'(x^2) + 4x^3 g''(x^2)}$$

§2.8 Related Rates.

#19 §2.8



Altitude = h is increasing @ 1 cm/min

Area = A is increasing @ 2 cm²/min

What's the rate of change in base = b when $h = 10$ cm & $A = 100$ cm²

Want $\frac{db}{dt}$

$h=10$
$A=100$

$$A = \frac{1}{2}bh$$

$$\frac{dA}{db} = \frac{1}{2}h \cdot \frac{dh}{db}$$

Assuming $h = h(b)$ is func. of b. Don't see $\frac{db}{dt}$ poppin' out.

Sep 19-12:15 PM

Try differentiating wrt t = time

$$\frac{dA}{dt} = \frac{1}{2}\left[\frac{db}{dt}\cdot h + b\cdot\frac{dh}{dt}\right]$$

$$f'g + fg'$$

$$2 = \frac{1}{2}\left[\frac{db}{dt}\cdot 10 + b\cdot 1\right]$$

Need b & then can find $b' = \frac{db}{dt}$

$$= \frac{1}{2}\left[10b' + 20\right]$$

$$A = \frac{1}{2}bh$$

$$100 = \frac{1}{2}b\cdot 10 = 5b$$

$$20 = b$$

$$2 = 5b' + 10$$

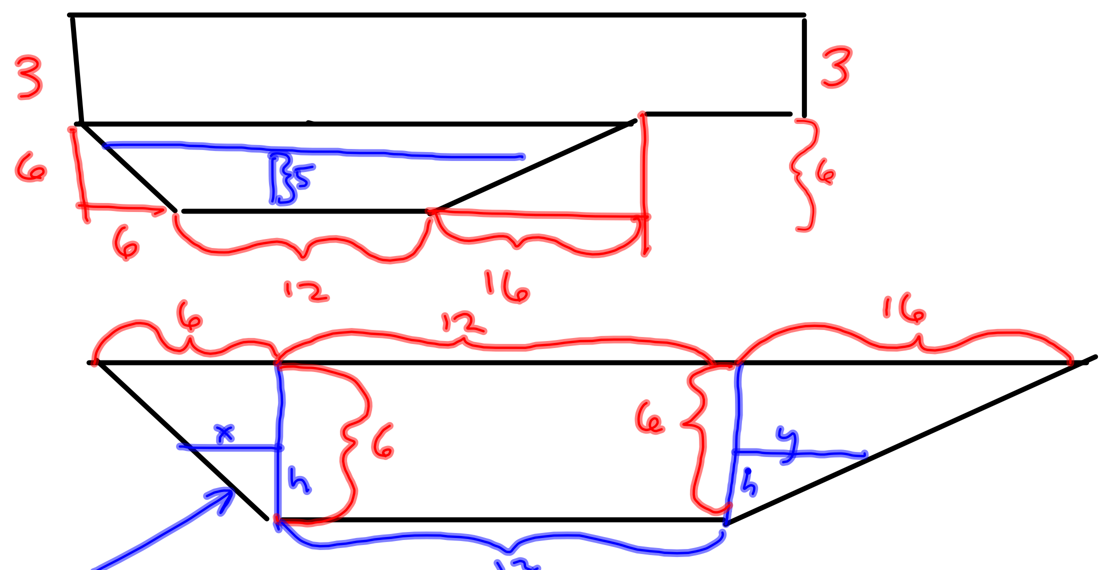
$$-8 = 5b'$$

$$-1.6\,\tfrac{cm}{min} = -\frac{8}{5} = b'$$

So it's getting taller so quickly, that the only way to keep $\frac{dA}{dt}$ down @ $2\,\frac{cm^2}{min}$ is if the base is actually shrinking.

$\frac{dV}{dt} = .8 \ ft^3/min$

How fast is depth increasing when the depth @ deepest point is 5 ft?



V = cross-sectional area times width.

= trapezoid area times 20'

$= \frac{1}{2}(b_1 + b_2) \cdot h \cdot 20$

h = height of water

$= \frac{1}{2}(12 + b_2)\cdot h \cdot 20$ b_2 is changing as h changes

$b_2 = 12 + x + y = 12 + \frac{8}{3}h + h = \boxed{\frac{11}{3}h + 12}$

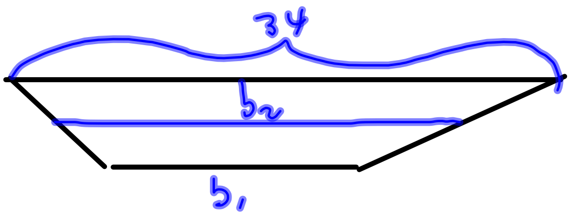
$\frac{h}{x} = \frac{6}{6} \Rightarrow h = x$, i.e., $\boxed{x = h}$

$\frac{h}{y} = \frac{6}{16} = \frac{3}{8} \Rightarrow h = \frac{3}{8}y \Rightarrow \boxed{y = \frac{8}{3}h}$

So $V = \frac{1}{2}\left(12 + \frac{11}{3}h + 12\right)\cdot h \cdot 20$

$= 10\left(\frac{11}{3}h + 24\right)h$

$V = 10h\left(\frac{11}{3}h + 24\right)$ want $\left.\frac{dh}{dt}\right|_{h=5}$



$$V = \frac{1}{2}(b_1+b_2)h \cdot 20$$
$$= \frac{1}{2}(12+b_2)\cdot 20h$$
$$= 10h(b_2+12)$$

b_2 @ bottom : $h=0$, $b_2=12$ $(0,12)$

b_2 @ top : $h=6$, $b_2=34$ $(6,34)$

looks like linear growth (both sides are sloping lines)

Build line $b_2 = mh+c$ as func. of h. $y=mx+b$, where $x=h=$depth.

$$m = \frac{34-12}{6-0} = \frac{22}{6} = \frac{11}{3}$$

$y - y_1$

$$b_2 = \frac{11}{3}(h-0)+12 = \frac{11}{3}h + 12$$
$$= m(x-x_1)+y_1$$
$$= m(h-h_1)+b_2\Big|_{h=h_1}$$

$$V = 10h\left(\frac{11}{3}h + 12 + 12\right) = 10h\left(\frac{11}{3}h + 24\right)$$

$$V = 10h\left(\frac{11}{3}h + 12 + 12\right) = 10h\left(\frac{11}{3}h + 24\right)$$

$$= \frac{110}{3}h^2 + 240h$$

$$\frac{dV}{dt} = \frac{220}{3}h \cdot \frac{dh}{dt} + 240\frac{dh}{dt}$$

Chain Rule on 'h', when differentiating wrt 't'.

$$.8 = \frac{220}{3}(5) \cdot \frac{dh}{dt} + 240\frac{dh}{dt}$$

$$.8 = \frac{1100}{3}h' + 240h' = \frac{4}{5} = \frac{1100 + 720}{3}h' = \frac{1820}{3}h'$$

$$\frac{4}{5} = \frac{1820}{3}h'$$

$$\frac{3}{1820} \cdot \frac{4}{5} = \frac{12}{9100} = \frac{6}{4550} = \frac{3}{2275} \ \frac{ft}{min}$$

Slooooowwww

These models are tough but REAL.

$$\frac{3}{2100}$$