

(M)  
§2.3#19

$$(x+x^{-1})^3 = x^3 + 3x^2x^{-1} + 3x \overset{x \cdot x^{-2}}{(x^{-1})^2} + (x^{-1})^3$$

$$\begin{array}{cccc} & & & 1 \\ & & & | \\ & & 1 & 1 \\ & & | & | \\ & 1 & 2 & 1 \\ & | & | & | \\ 1 & 3 & 3 & 4 \\ | & & & | \end{array}$$

$$= x^3 + 3x + 3x^{-1} + x^{-3} \Rightarrow$$

$$\frac{d}{dx} (x+x^{-1})^3 = 3x^2 + 3 - 3x^{-2} - 3x^{-4}$$

Chain Rule way:  $\frac{dy}{dx} = 3(x+x^{-1})^2(1-x^{-2})$

2.3 way,  
using product  
Rule

$$(fg)' = f'g + fg'$$

$$(fgh)' = f'gh + fg'h + fgh'$$

$$\begin{aligned} \text{Pf } (f(gh))' &= f'(gh) + f(gh)' \\ &= f'gh + f(g'h + gh') \\ &= f'gh + \underline{fg'h} + fgh' \end{aligned}$$

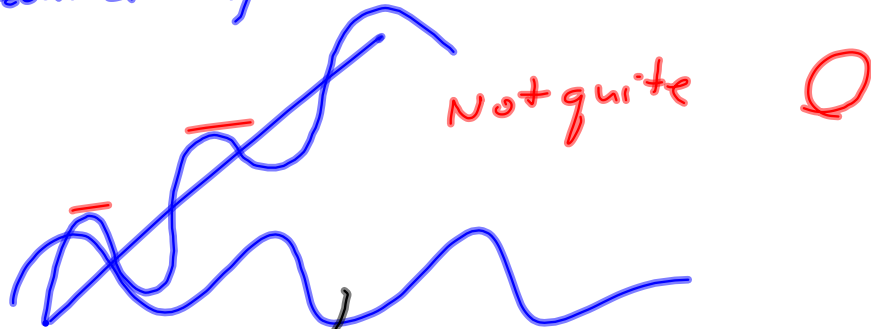
$$y = (x+x^{-1})^3 = (x+x^{-1})(x+x^{-1})(x+x^{-1})$$

$$\begin{aligned} \Rightarrow y' &= (1-x^{-2})(x+x^{-1})^2 + \underline{(x+x^{-1})(1-x^{-2})(x+x^{-1})} \\ &\quad + (x+x^{-1})^2(1-x^{-2}) \\ &= 3(1-x^{-2})(x+x^{-1})^2 \end{aligned}$$

2.4 #33

Similar Question:

For what values of  $x$  does  $f(x) = x + 2\cos x$  have a horizontal tangent.



→ Slope's negative whenever  $2\sin x > 1$

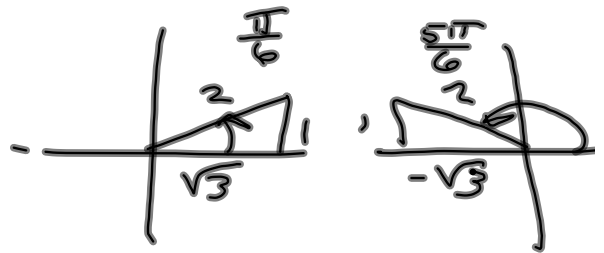
$$y' = 1 - 2\sin x \stackrel{\text{SET}}{=} 0 \rightarrow$$

$$-2\sin x = -1 \rightarrow$$

$$\sin x = \frac{1}{2}$$

$$\frac{\pi}{6} + 2n\pi$$

$$\frac{5\pi}{6} + 2n\pi$$



$$\left\{ x \mid x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z} \right\}$$

$n \in \mathbb{Z}$  means  $n = 0, \pm 1, \pm 2, \pm 3, \dots$

## §2.7 Boyle's Law Adiabatic

If Temp held constant, then  
 $PV = \text{Constant}$ .

(a) Find rate of change of volume wrt P.

$$\frac{dV}{dP}$$

My 1<sup>st</sup> thought: Hey! §2.6

$$PV = C$$

$$P'V + PV' = 0$$

$$PV' = -P'V$$

$$V' = -\frac{P'V}{P} = -\frac{P'}{P} \cdot \frac{C}{P} = -\frac{P'C}{P^2}$$

Differentiating  
 wrt time.

(Assume  $P = P(t)$ )

If we follow instructions (I didn't),  
 we differentiate wrt P, then

$P' = 1$ , which reduces to

$$V' = -\frac{C}{P^2}$$

Book wants:

$$PV = C$$

$$\Rightarrow V = \frac{C}{P} = CP^{-1}$$

$$\Rightarrow \frac{dV}{dP} = -1CP^{-2} = -\frac{C}{P^2} = \frac{dV}{dP}$$

My approach was slightly more general.

#235 "steadily compressed" means.  
change in volume.  $\frac{dP}{dt} = \text{Constant}$  was my thought.

"Steadily compress" for 10 min.

Is volume decreasing more rapidly at start or at the end?  $\rightarrow \frac{dV}{dt}$

$$\frac{dV}{dP} = -\frac{c}{P^2}$$

$P \uparrow$  increases  $\rightarrow$  increases

Decreasing in absolute value

$|\frac{c}{P^2}|$  getting smaller

It was shrinking faster at the start.

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(c) Isothermal Compressibility

$\beta = \frac{1}{P}$  we want to show.

$\beta = -\frac{1}{V} \cdot \frac{dV}{dP}$  from example 5.

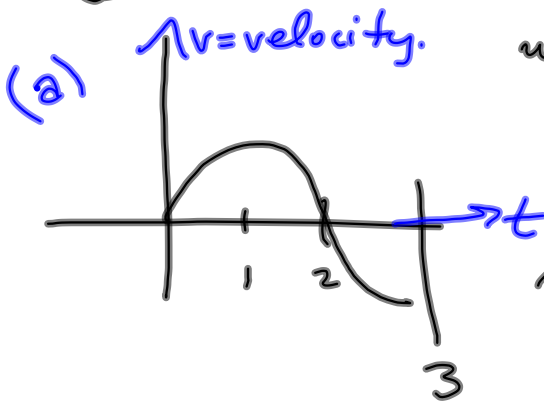
$$V = \frac{c}{P}$$

$$\frac{dV}{dP} = -\frac{c}{P^2}$$

$$= -\frac{1}{\frac{c}{P}} \cdot -\frac{c}{P^2} = -\frac{P}{c} \cdot -\frac{c}{P^2} = \frac{1}{P}$$

§2.7 #s 5, 6

⑤ Graphs of VELOCITY funcs.



when is it slowing-down?

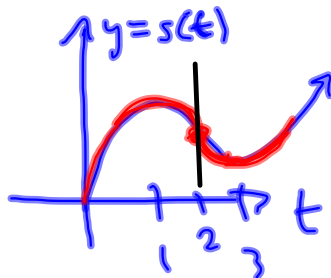
$$\text{when } \frac{dv}{dt} < 0$$

(1, 3) is where  $v$  has negative slope.

speeding-up?

(0, 1)

⑥ Graphs of POSITION func,  $s(t)$

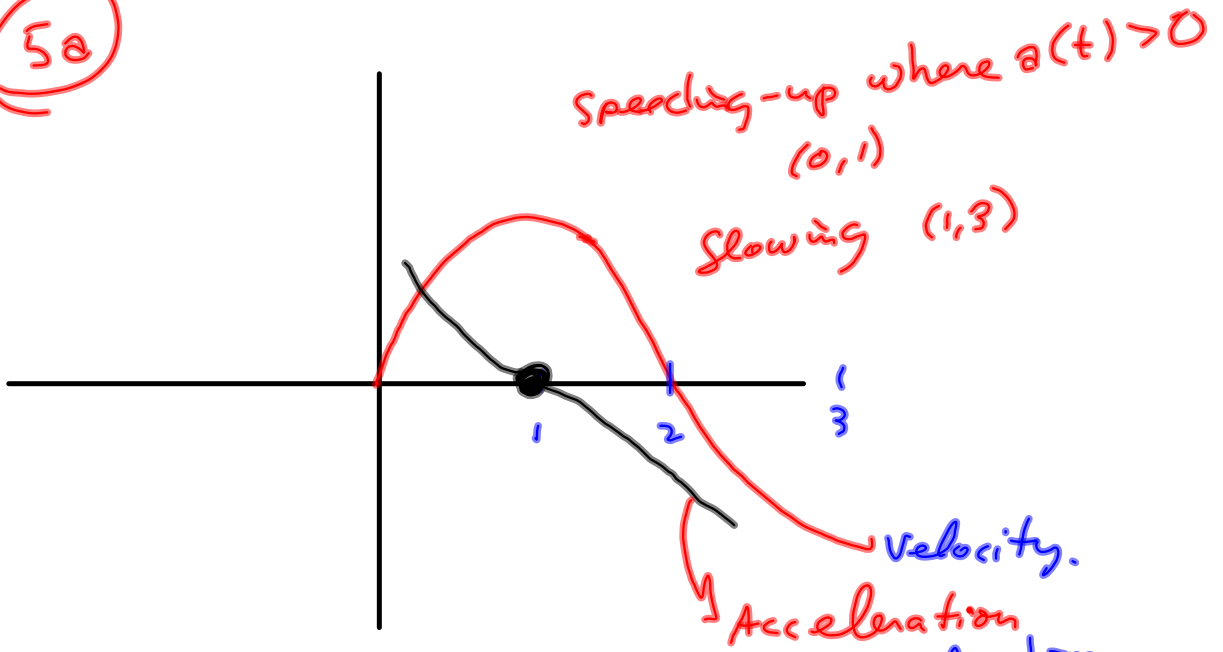


when's it speeding up?

(2, 4)

slowing: (0, 2)

5a



6a

