§2.5 Chain Rule

Test towards end of next week. Thurs/Fri.

§2.6 Implicit Differentiation

#s 1-27, 45, 47

§2.7 Derivatives in science

#s 5, 6, 8, 11, 15

§2.8 Related Rates

#s 3-17, 23

§2.9 Linear Approximation and Differentials.

Sep 16-12:07 PM

Recall Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$$\frac{d}{dx}\left[\cos(3x^2-5)\right] = \left(-\sin(3x^2-5)\right) \cdot (6x)$$

$$= -6x\sin x\,(3x^2-5)$$

Please differentiate wrt x :

$$x f(x) \xrightarrow{\frac{d}{dx}} 1 f(x) + x f'(x)$$

$$\frac{d}{dx}\left[x\, f(x)^2\right] = 1(f(x))^2 + x \cdot 2f(x) \cdot f'(x)$$

$$\frac{d}{dx}\left[x\, y^2\right] = y^2 + x \cdot 2yy'$$ if we assume y depends on x.

$$\frac{d}{dx}\left[x^2y + xy^3 - 5x^2y^2\right] \qquad \boxed{\frac{dy}{dx} = y'}$$

$= 2xy + x^2y' + y^3 + 3xy^2y' - 10xy^2 - 10x^2yy'$

$= 2xy + x^2y' + y^3 + x \cdot 3y^2y' - 10xy^2 - 5x^2 2yy'$

$n=1$ $\quad$ $n=3$ $\quad$ $n=2$

$$\frac{d}{dx}\left[x^2\right] = 2xx' = 2x$$

$$\frac{dx}{dx}$$

$$\frac{d}{dx}\left[f(x)^n\right] = n\, f(x)^{n-1} \cdot f'(x)$$

Find $\frac{dy}{dx}$ if $\qquad \left(y' = \frac{dy}{dx}\right)$

$$x^3 + x^2y + xy^2 - y^3 = 2,100,500,372\pi$$

* $3x^2 + 2xy + x^2y' + y^2 + 2xyy' - 3y^2y' = 0$

$x^2y' + 2xyy' - 3y^2y' = -3x^2 - 2xy - y^2$

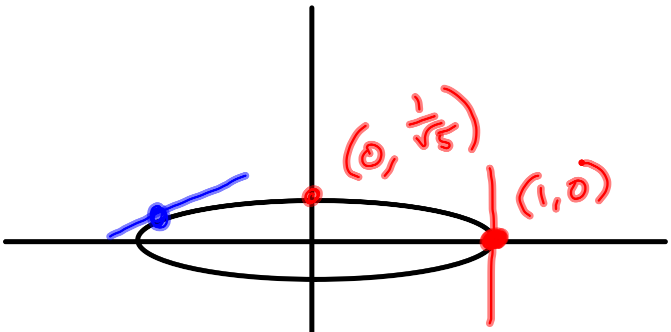
$y'(x^2 + 2x - 3y^2) = -3x^2 - 2xy - y^2$

$$y' = \frac{-3x^2 - 2xy - y^2}{x^2 + 2xy - 3y^2}$$

$(fg)' = f'g + fg'$

$$x^2 + 5y^2 = 1$$

$$x^2 + \frac{y^2}{\frac{1}{5}} = 1$$



without §2.6:

Find eq'n of tangent lines to the ellipse

@ $x=1$

§2.6 way:

$$2x + 10yy' = 0$$

$$10yy' = -2x$$

$$y' = -\frac{2x}{10y} = -\frac{1}{5}\frac{x}{y}$$

$$5y^2 = 1 - x^2$$

$$y^2 = \frac{1-x^2}{5}$$

$$y = \pm\sqrt{\frac{1-x^2}{5}} = \left(\frac{1-x^2}{5}\right)^{\frac{1}{2}}, \quad -\left(\frac{1-x^2}{5}\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left(\frac{1-x^2}{5}\right)^{-\frac{1}{2}}\left(-\frac{2x}{5}\right)$$

@ $x=1$, we have

$$y' = \frac{1}{2}\left(\frac{0}{5}\right)^{-\frac{1}{2}}\left(-\frac{2(1)}{5}\right)$$

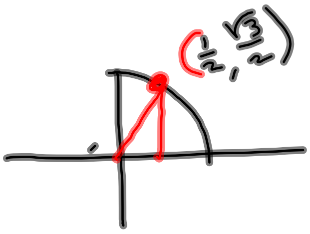
Circles:

$x^2+y^2=1$

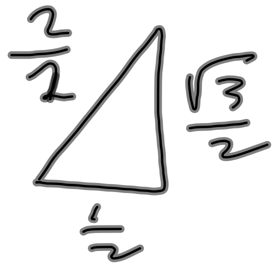
$y^2=1-x^2$

$y=\pm\sqrt{1-x^2} = $ → $(1-x^2)^{\frac{1}{2}}$ Top

→ $(1-x^2)^{-\frac{1}{2}}$ Bottom



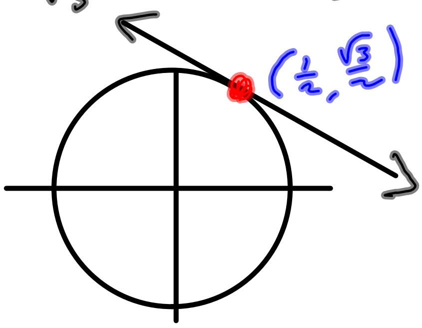
Tangent line to the circle @ $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

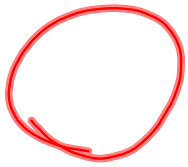


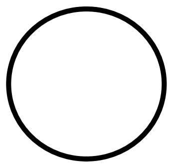
$y' = \text{slope} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) = -\frac{x}{(1-x^2)^{\frac{1}{2}}}$

$$m = y'\left(\tfrac{1}{2}\right) = -\frac{\frac{1}{2}}{\left(1-\frac{1}{4}\right)^{1/2}} = \frac{-\frac{1}{2}}{\left(\frac{3}{4}\right)^{1/2}} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$y = -\frac{1}{\sqrt{3}}\left(x-\frac{1}{2}\right) + \frac{\sqrt{3}}{2}$$







2.6 way: $x^2+y^2=1 \longrightarrow$

$2x+2yy'=0 \longrightarrow$

$yy'=-x \Longrightarrow$

$y' = -\dfrac{x}{y} \Longrightarrow y'\left(\tfrac{1}{2}\right) = -\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= -\dfrac{1}{\sqrt{3}} = m$

$y = -\dfrac{1}{\sqrt{3}}\left(x-\dfrac{1}{2}\right) + \dfrac{\sqrt{3}}{2}$