2.4	#s 1 – 31, 55, 70, 73	Trigs
2.5	#s 3 – 25,31, 33, 35, 39, 43, 52B	Chain

Typo from yesterday:
2.4 & 2.5 assignments are backwards!
will fix after class

Test prep: On homework, esp. §2.4,
you see me circle early answer & say STOP!
Don't simplify unless specifically
required.

§2.5

$$\frac{d}{dx}\left[\text{Outer}(\text{inner})\right] = \frac{d\text{ outer}}{d\text{ inner}} \cdot \frac{d\text{ inner}}{d\,x}$$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$$\frac{d}{dx}\left[(x^2-3x)^{35}\right] = \underbrace{35(x^2-3x)^{34}}_{\frac{df}{dg}}\underbrace{(2x-3)}_{\frac{dg}{dx}}$$

$g(x) = x^2-3x$

$f(g) = g^{35}$

$f'(g) = 35g^{34}$

$g'(x) = 2x-3$

$\boxed{E}$ I make \$5 profit off every shirt.

Let g = the # of shirts, and

$f(g) = 5g$ = profit from shirts.

Suppose I'm making 20 shirts per hour.

Let x = time, in hours.

Then $g(x) = 20x$ = the # of shirts I make in x hours.

So profit, as a function of time, is $f(g(x)) = f(20x) = 5 \cdot 20x = 100x$.

So Profit/hour is $\frac{d}{dx}[100x] = 100\ \$/hr$

$$\left(5 \frac{\$}{\text{shirt}}\right)\left(\frac{20 \text{ shirts}}{\text{hr}}\right) = 100 \frac{\$}{\text{hr}}.$$

Chain Rule Says

$f(g) = 5g \qquad \frac{df}{dg} = 5$

$g(x) = 20x \qquad \frac{dg}{dx} = 20$

$\therefore \frac{df}{dx} = 5 \cdot 20 = 100.$

$$\frac{d}{dx}\left[\cos(27x^2-5x)\right] = \left[-\sin(27x^2-5x)\right](54x-5)$$

$f(g) = \cos$

Issues with ambiguities #s 3, 13, 15, 31

#3 $y = \tan \pi x$ should be $\tan(\pi x)$

#13 $y = \cos(a^3 + x^3)$ "Take the derivative" is unclear. What's the independent variable? 'a' or 'x'?

Assume it's 'x': we're doing $\frac{dy}{dx}$, here.

15 $y = x \sec kx$ should be $x\sec(kx)$

31 $y = \sin(\tan 2x)$ should be

$\sin(\tan(2x))$

§2.3 #69

$$\frac{d}{dx}\left[h(x)g(x)\right] = h'(x)g(x) + h(x)g'(x)$$

$\sqrt{x} = x^{\frac{1}{2}}$ $\quad \frac{1}{2}x^{-\frac{1}{2}}g(x) + x^{\frac{1}{2}}g'(x)$

$h'(x) = \frac{1}{2}x^{-1/2}$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{(g(x))^2}$$

§2.3
89

$y = f(x) = ax^3 + bx^2 + cx + d$

know $f(2) = 0$

$f(-2) = 6$

$f'(2) = f'(-2) = 0$ Find a, b, c, d.

$f'(2) = f'(-2) = 0$ route:

$f'(x) = 3ax^2 + 2bx + c = 3a(x-2)(x+2)$

$= 3a(x^2 - 4)$

$= 3ax^2 - 12a$

$\Rightarrow b = 0$

$3ax^2 = 3ax^2$ ✓

$c = -12a$

$f(x) = ax^3 - 12ax + d$

$f(2) = 0 \Rightarrow 8a - 24a + d = 0$

$-16a + d = 0$

$d = 16a$

$ax^3 - 12ax + 16a$

$f(-2) = 6$: $-8a + 24a + 16a = 6$

$32a = 6$

$a = \frac{6}{32} = \boxed{\frac{3}{16} = a}$

$c = -12a = -\frac{36}{16} = -\frac{18}{8} = -\frac{9}{4}$

$$\boxed{\frac{3}{16}x^3 - \frac{9}{4}x + 3}$$

~~Book sol.~~ My online solution is wrong

§2.4 Examples

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

(11) $f(\theta) = \dfrac{\sec\theta}{1+\sec\theta}$

$$\Rightarrow f'(\theta) = \frac{(\sec\theta\tan\theta)(1+\sec\theta) - (\sec\theta)(\sec\theta\tan\theta)}{(1+\sec\theta)^2}$$

ON TEST, STOP!

ON Homework, it's good practice to try & manipulate the expression to look like the book answer.

$$= \frac{\sec\theta\tan\theta + \sec^2\theta\,(\tan\theta) - \sec^2\theta\tan\theta}{(1+\sec\theta)^2}$$

I don't see this being simplified

$$= \frac{\sec\theta\tan\theta + \sec^2\theta\tan\theta - \sec^2\theta\tan\theta}{(1+\sec\theta)^2}$$

$$= \frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}$$

(11) $f(\theta) = \frac{\sec\theta}{1+\sec\theta} \cdot \frac{1-\sec\theta}{1-\sec\theta} = \frac{\sec\theta - \sec^2\theta}{1-\sec^2\theta}$

$= \frac{\sec\theta - \sec^2\theta}{-\tan^2\theta} = \frac{\sec^2\theta}{\tan^2\theta} - \frac{\sec\theta}{\tan^2\theta}$

$= \frac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} - \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{1}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\cos\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}$

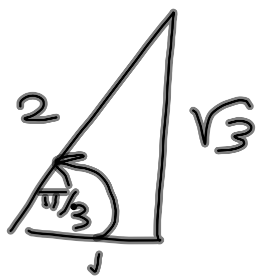
$= \csc^2\theta - \csc\theta\cot\theta$ Just to show the different forms you can get.

2.4 #21 tan. line @ (x_1, y_1) when

$y = \sec x$ @ $\left(\frac{\pi}{3}, 2\right)$

$y' = \sec x \tan x \Longrightarrow$

$y'\left(\frac{\pi}{3}\right) = \sec\frac{\pi}{3}\tan\frac{\pi}{3} = 2\cdot\sqrt{3}$



$y = 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 2$

$y = m(x - x_1) + y_1$

Normal Line $m_\perp = -\frac{1}{m}$