§2.3 #s 77, 79

$ax^2+bx+c=0 \Longrightarrow$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{\text{discriminant}}}{2a}$$

$b^2-4ac<0 \Longrightarrow$ No real solns.

78 Eq'n of tan. line to $y = x\sqrt{x}$, that's $\parallel$ to $y=3x+1$.

$y = x\sqrt{x} = x \cdot x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}}$

$\Longrightarrow y' = \frac{3}{2}x^{\frac{1}{2}}$ want $= m = 3$

Need x_1 for (x_1, y_1) for $y = m(x-x_1)+y_1$

Solve $\frac{3}{2}x^{\frac{1}{2}} = 3 \Longrightarrow$

$\frac{2}{3}\cdot\frac{3}{2}x^{\frac{1}{2}} = \frac{3}{1}\cdot\frac{2}{3} \Longrightarrow$

$4^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = \left((4)^{\frac{1}{2}}\right)^3 = 2^3$

$x^{\frac{1}{2}} = 2 \Longrightarrow$

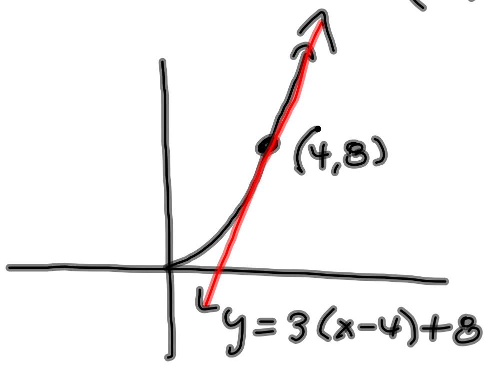
$\left(x^{\frac{1}{2}}\right)^2 = 2^2$

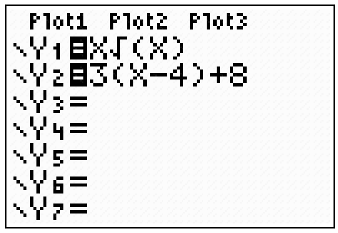
$x = 4 \Longrightarrow y = 4^{3/2} = 2^3 = 8$

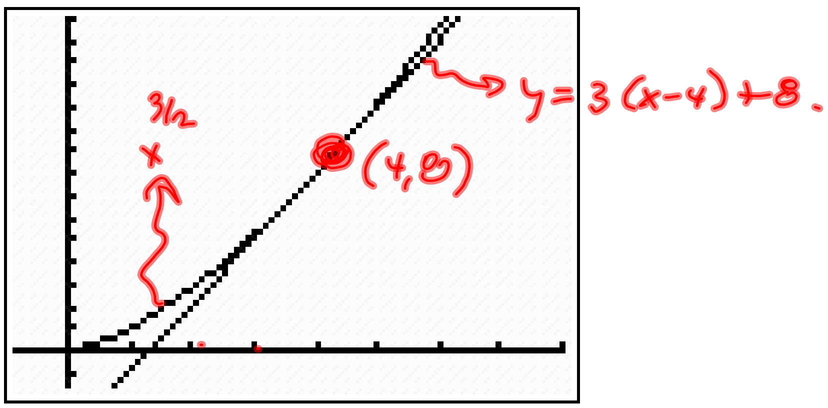
$(x_1, y_1) = (4, 8)$ &

$$\boxed{y = 3(x-4)+8}$$

$x\sqrt{x} = x^{3/2} = \left(x^{\frac{1}{2}}\right)^3$







2.4	#s 1 – 31, 55, 70, 73	Trigs
2.5	#s 3 – 25,31, 33, 35, 39, 43, 52B	Chain

§2.4 $\frac{d}{dx}[\sin x] = \cos x$

Proof

$$\frac{f(x+h)-f(x)}{h} = \frac{\sin(x+h)-\sin x}{h}$$

$$= \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \frac{\sin x[\cos h - 1] + \sin h \cos x}{h}$$

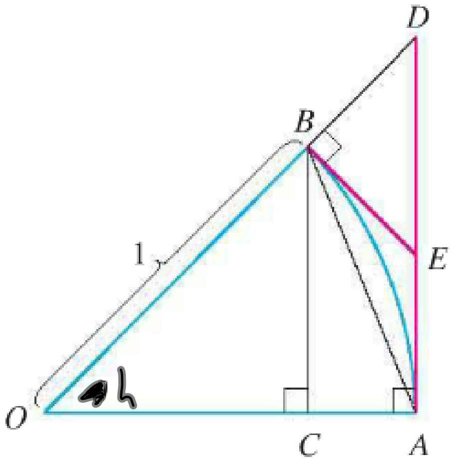
$$= \frac{\cos h - 1}{h}\sin x + \frac{\sin h}{h}\cos x$$

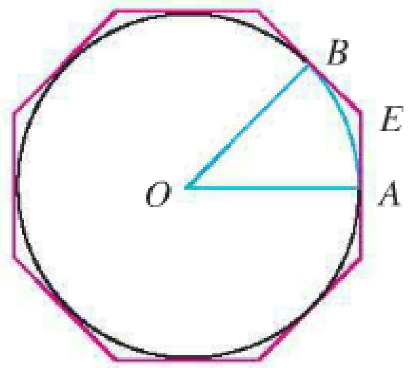
$\rightarrow 0$ as $h \to 0$ is the goal.

$\rightarrow 1$ as $h \to 0$

Sep 11-12:02 PM

Recall arc length $= r\theta = h$, since $r=1$, $\theta = h$ = arc AB



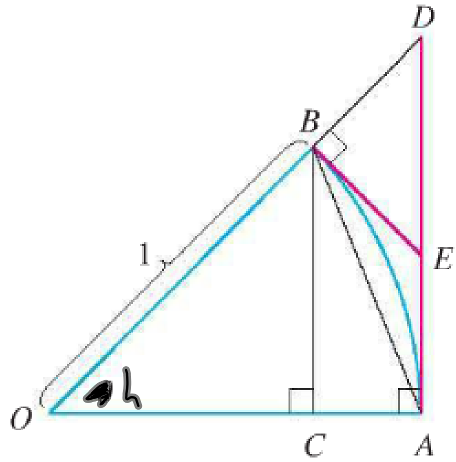


$$|BC| < |AB| < \text{arc } AB = h$$

$$\sin h < h$$

$$\boxed{\frac{\sin h}{h} < 1}$$

Sep 11-10:17 AM



$h = \text{arc}AB < |AE| + |EB|$

$< |AE| + |ED|$

$= |AD|$, i.e.

$h < |AD|$

Notice $\dfrac{|AD|}{|OA|} = \tan h$

i.e., $h < \tan h$, since $|OA| = 1$

$h < \tan h$

$h < \dfrac{\sin h}{\cos h}$

$$\boxed{\cos h < \frac{\sin h}{h}}$$

Put it together:

$$\cos h < \frac{\sin h}{h} < 1$$

$h \to 0$: $\cos h \to 1$; $h \to 0$: $1 \to 1$

Squeeze it!

$$\boxed{\lim_{h \to 0} \frac{\sin h}{h} = 1}$$

$$\frac{\cos h - 1}{h} = \frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1} = \frac{\cos^2 h - 1}{h(\cos h + 1)}$$

$$= \frac{-\sin^2 h}{h(\cos h + 1)} = -\frac{\sin h}{h} \cdot \frac{\sin h}{\cos h + 1}$$

$$\xrightarrow{h \to 0} -1 \cdot \frac{0}{1+1} = -1 \cdot 0 = 0.$$

Back to main proof, we now KNOW

$$\frac{\cos h - 1}{h} \xrightarrow{h \to 0} 0 \;\&\; \frac{\sin h}{h} \xrightarrow{h \to 0} 1, \Longrightarrow$$

$$\frac{\cos h - 1}{h} \cdot \sin x + \frac{\sin h}{h} \cdot \cos x \xrightarrow{h \to 0} \underline{\cos x} \; \blacksquare$$

Sep 11-12:29 PM

See Table, pg 144. You should memorize it, but you can reconstruct it by knowing $\frac{d}{dx}[\sin x] = \cos x$

#17 $\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right]$

#18 $\frac{d}{dx}[\sec x]$

#19 $\frac{d}{dx}[\cot] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right]$

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$$\boxed{\frac{d}{dx}[\cos x] = -\sin x}$$

Use this and the result for $\frac{d}{dx}[\sin x]$ for #s 17, 19. No $h \to 0$ needed.

Pf

$$\frac{\cos(x+h) - \cos x}{h} = \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$\frac{\cos h - 1}{h}\cos x - \frac{\sin h}{h}\sin x \xrightarrow{h \to 0} -\sin x \quad \blacksquare$$