§1.7 #32 $\lim_{x\to 2} x^3 = 8$

Scratch

want $|x^3-8| < \varepsilon$

$$-\varepsilon < x^3-8 < \varepsilon$$

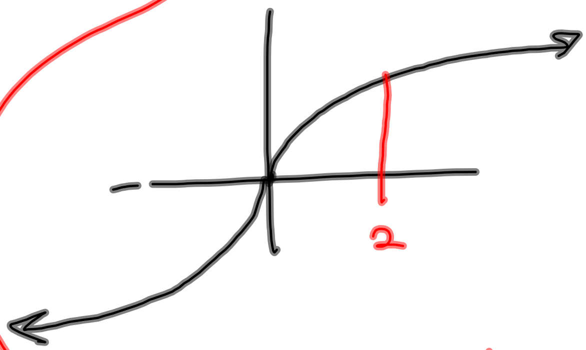
$$8-\varepsilon < x^3 < 8+\varepsilon$$

$$\sqrt[3]{8-\varepsilon} < x < \sqrt[3]{8+\varepsilon}$$

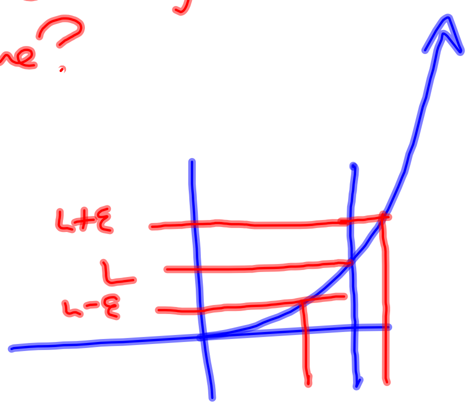
So what's that say about x close to x=2?

$$\boxed{\sqrt[3]{8-\varepsilon} - 2} < x-2 < \boxed{\sqrt[3]{8+\varepsilon} - 2}$$

$\delta = \sqrt{8+\varepsilon} - 2$



which is greater in absolute value?



§2.2 #24

$\frac{d}{dt}\left[\frac{1}{\sqrt{t}}\right]$ Find $\mathcal{D}(g), \mathcal{D}(g')$

$g(t) = \frac{1}{\sqrt{t}}$

§2.3 Quickie

$$\frac{d}{dt}\left[\frac{1}{\sqrt{t}}\right] = \frac{d}{dt}\left[t^{-\frac{1}{2}}\right] = -\frac{1}{2}t^{-\frac{3}{2}} = \boxed{-\frac{1}{2\sqrt{t^3}}}$$

$$\frac{g(t+h)-g(t)}{h} = \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} = \frac{1}{h}\left[\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}\right]$$

$$= \frac{1}{h}\left[\frac{1}{\sqrt{t+h}}\cdot\frac{\sqrt{t}}{\sqrt{t}} - \frac{1}{\sqrt{t}}\cdot\frac{\sqrt{t+h}}{\sqrt{t+h}}\right]$$

$$\boxed{\begin{aligned}&(\sqrt{t})^2-(\sqrt{t+h})^2\\ &= t-(t+h)\\ &= t-t-h\end{aligned}}$$

$$= \frac{1}{h}\left[\frac{\sqrt{t}-\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}\right]\left[\frac{\sqrt{t}+\sqrt{t+h}}{\sqrt{t}+\sqrt{t+h}}\right]$$

$$= \frac{1}{h}\left[\frac{t-(t+h)}{\sqrt{t}\sqrt{t+h}(\sqrt{t}+\sqrt{t+h})}\right] = \frac{-1}{\sqrt{t}\sqrt{t+h}(\sqrt{t}+\sqrt{t+h})}$$

$$\xrightarrow{h\to 0} \frac{-1}{\sqrt{t}\sqrt{t}(\sqrt{t}+\sqrt{t})} = -\frac{1}{t(2\sqrt{t})}$$

$$= \boxed{-\frac{1}{2t\sqrt{t}}} = -\frac{\sqrt{t}}{2t^2}$$

§2.3 #19-ish.

$$h(x) = \frac{3x^3+5x^2-11x+7}{\sqrt{x}} = \frac{f}{g}$$

$$\Rightarrow h'(x) = \left(\frac{f}{g}\right)'(x) = \frac{f'g-fg'}{g^2} \text{ is harder}$$

than: $3x^{5/2}+5x^{3/2}-11x^{1/2}+7x^{-1/2} = h(x)$

$$\boxed{\lim_{x\to 2} \frac{x^2+x-6}{|x-2|} \quad \not\exists}$$

$$|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 < 0 \end{cases}$$

$$= \begin{cases} x-2 & \text{if } x \geq 2 \\ -(x-2) & \text{if } x < 2 \end{cases}$$

$$\lim_{x\to 2^+} \frac{x^2+x-6}{|x-2|} = \lim_{x\to 2^+} \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2^+} (x+3) = +5$$

$$\lim_{x\to 2^-} \frac{(x+3)\cancel{(x-2)}}{-\cancel{(x-2)}} = \lim_{x\to 2^-} (-(x+3)) = -5$$

Sep 10-12:49 PM