§2.1 #8

Find eq'n of tangent line to

$f(x) = \frac{2x+1}{x+2}$ @ $(1,1)$

Goal:

$y = f'(1)(x - x_1) + y_1$

$= f'(1)(x-1) + 1$

Need this.

Rather do it 2.2/2.3 way

§2.1 way :

$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ =

$$\frac{f(1+h) - f(1)}{h} = \frac{\frac{2(1+h)+1}{(1+h)+2} - \frac{2(1)+1}{1+2}}{h}$$

$$= \frac{\frac{2+2h+1}{1+h+2} - \frac{2+1}{3}}{h} = \frac{\frac{3+2h}{3+h} - \frac{3}{3}}{h} = \frac{\frac{3+2h}{3+h} - \frac{1}{1} \cdot \frac{3+h}{3+h}}{h}$$

$$= \frac{\frac{3+2h-(3+h)}{3+h}}{h} = \frac{\frac{3+2h-3-h}{3+h}}{h} = \frac{\frac{h}{3+h}}{h}$$

$$= \frac{1}{h}\left(\frac{h}{3+h}\right) = \frac{1}{3+h} \xrightarrow{h \to 0} \frac{1}{3} = f'(1) = m_{tan}$$

$$y = m(x - x_1) + y_1$$

$$= f'(1)(x - x_1) + y_1$$

$$\boxed{= \frac{1}{3}(x-1) + 1 = y}$$

§2.1 way :

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

§2.2 way:

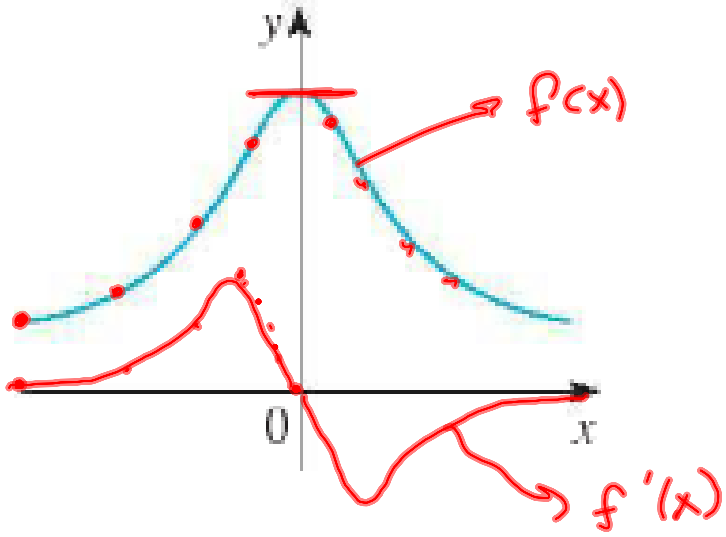
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Then plug in x=1 @ the end for f'(1)

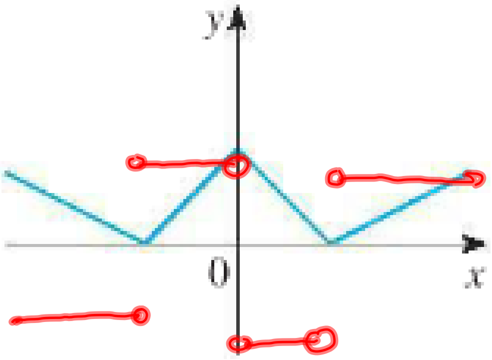
§2.3 way:

Learn the slick derivative rules.

6.



9.



The derivative of piecewise-linear function is a step function

2.1	#s 1, 3, 4, 7, 8, 11, 12, 13[3], 33, 35 – 53 and 54 are Bonus
2.2	#s 5 – 11 Odds, 19, 23 – 25 ALL 41, 43
2.3 I	#s 1 – 15 Odds, 17[4], 19[5], 23 – 35 Odds (Due Thursday)
2.3 II	#s 51, 52, 55, 57, 59, 63, 67, 69, 73, 75, 77, 79, 81, 89 (Due Friday)

[2] $FACT: x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + ... + x^2 + x + 1)$ e.g. $x^3 - 1 = (x-1)(x^2 + x + 1)$ This is a fun fact(orization) that we will use to prove our power rule for derivatives in a week or so.

[3] This one should look familiar.

[4] There's a quick way to work this, if you're good with Algebra.

[5] Same as #4, but it's even BETTER if you learn the Chain Rule, and apply IT to this power of a function. Chain Rule doesn't kick in until Section 2.5, ALAS!!! So let's talk about that one and leave it blank, with LOTS of room after it.

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§2.3 Differentiation Rules.

$\frac{d}{dx}[\text{thing}]$ = derivative with respect to x of thing.

$\frac{d}{dx}[x^n] = nx^{n-1}$ if $n \neq 0$. Power Rule.

$f(x) = x^3 \Longrightarrow f'(x) = 3x^2$

$f(x) = \frac{1}{\sqrt[5]{x^2}} = \frac{1}{x^{2/5}} = x^{-\frac{2}{5}} \Longrightarrow$

$f'(x) = -\frac{2}{5}x^{-\frac{7}{5}}$ ✓ $\left(= \frac{-2}{5\sqrt[5]{x^7}} \right)$

I like this

This is ok

$\frac{d}{dx}[f(x)+g(x)] = \boxed{(f+g)' = f'+g'}$ Sum Rule (Difference rule)

c is a constant $\Longrightarrow$

$\boxed{(cf)' = cf'}$

This makes it a linear operator. $\rightarrow$ Advanced Calculus.

$\frac{d}{dx}[c] = 0$

$f(x) = x^3 - 5x^2 + 3x + 2 \Longrightarrow$

$f'(x) = 3x^2 - \underline{10x} + 3$

$\rightarrow 5 \cdot 2x$

The toughies: I keep things alphabetical. Book doesn't.

$$(fg)' = f'g + fg'$$ PRODUCT RULE

$$\frac{d}{dx}\left[\underbrace{(x^2-7x)}_{f}\underbrace{(x^3+5x^2-3x+7)}_{g}\right]$$

$$= (2x-7)(x^3+5x^2-3x+7) + (x^2-7x)(3x^2+10x-3)$$

f' g $+$ f g'

$(fg)(x) = x^5 + 5x^4$ _ Too big. Hulk Tired.

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CAS confirmation.

$f := x \rightarrow (x^2 - 7 \cdot x) \cdot (x^3 + 5 \cdot x^2 - 7 \cdot x + 7)$

$$x \rightarrow (x^2 - 7x)(x^3 + 5x^2 - 7x + 7) \quad (1)$$

$fp := D(f)$

$$x \rightarrow (2x - 7)(x^3 + 5x^2 - 7x + 7) + (x^2 - 7x)(3x^2 + 10x - 7) \quad (2)$$

$expand(f(x))$

$$x^5 - 2x^4 - 42x^3 + 56x^2 - 49x \quad (3)$$

$g := x \rightarrow x^5 - 2x^4 - 42x^3 + 56x^2 - 49x$

$$x \rightarrow x^5 - 2x^4 - 42x^3 + 56x^2 - 49x \quad (4)$$

$gp := D(g)$

$$x \rightarrow 5x^4 - 8x^3 - 126x^2 + 112x - 49 \quad (5)$$

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$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\left[\frac{x^2+3x}{x^3-2}\right] = \frac{\overset{f'}{(2x+3)}\,\overset{g}{(x^3-2)} \overset{-}{-} \overset{f}{(x^2+3x)}\,\overset{g'}{(3x^2)}}{\underset{g^2}{(x^3-2)^2}}$$

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