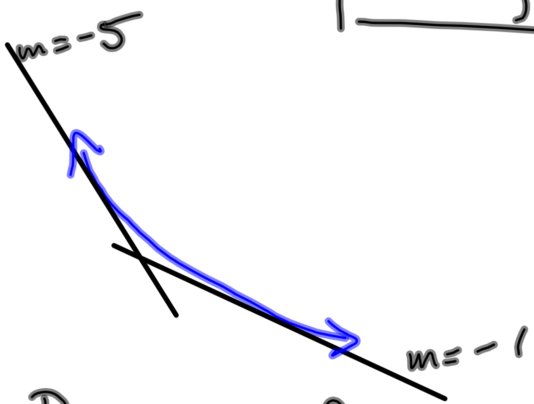
§2.1 #12

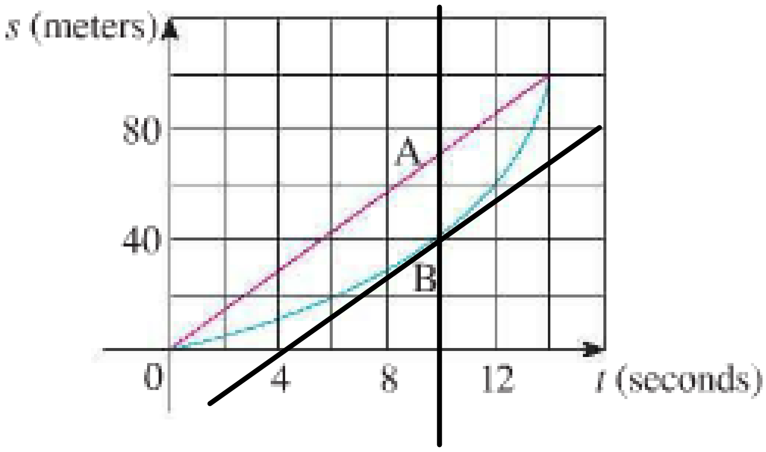
§1.7 #32

Harry S Mills



Decreasing function with increasing slope.

12. Shown are graphs of the position functions of two runners, .
and B, who run a 100-m race and finish in a tie.



§1.7 #32

Claim $\lim_{x \to 2} x^3 = 8$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$
$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Scratch
want $|x^3-8| < \varepsilon$ when $0 < |x-2| < \delta$

ε will be given. We find δ.

$|x^3-8| = |x^3-2^3| = |x-2||x^2+2x+4| < \varepsilon$

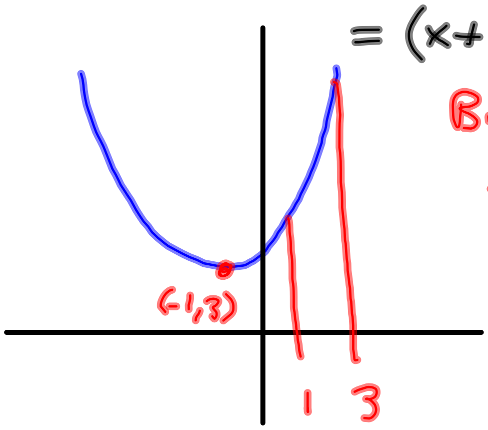
is gonna be $< \delta \boxed{|x^2+2x+4|} \overset{\text{want}}{<} \varepsilon$

Thought process: How big can $|x^2+2x+4|$ get, if x is close to 2?

Assume $\delta \leq 1 \implies$

$1 < x < 3$

Have a look @ x^2+2x+4
$= x^2+2x+1-1+4$
$= (x+1)^2+3$



Biggest @ $x=3$:
$3^2+2(3)+4 = \boxed{19}$

Now we can write the proof!

$\ldots \ \delta|x^2+2x+4|$
$< \delta \cdot 19 \overset{\text{want}}{\leq} \varepsilon$

$\delta \leq \frac{\varepsilon}{19}$

<u>Proof</u> Let $\varepsilon > 0$ be given. Define $\delta = \min\left\{1, \frac{\varepsilon}{19}\right\}$. Then if $0 < |x-2| < \delta$, we have

$$
\begin{aligned}
|x^3 - 8| &= |x-2|\,|x^2 + 2x + 2^2| \\
&< \delta\,|x^2 + 2x + 4| \\
&\leq \delta \cdot 19 \\
&\leq \frac{\varepsilon}{19} \cdot 19 = \varepsilon \quad \blacksquare
\end{aligned}
$$

A few things about "Δ" means change

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x) = \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

= the derivative of y with respect to x.

= the slope function for $y=f(x)$.

The variable with respect to which we're differentiating.

$\frac{dy}{dx}$ = Liebniz Notation.

we will use it when it's more convenient

$f'(x)$ Doesn't always exist.

If it does, then $f(x)$ is continuous.

If $f'(x)$ exists, then $f(x)$ is "smooth"

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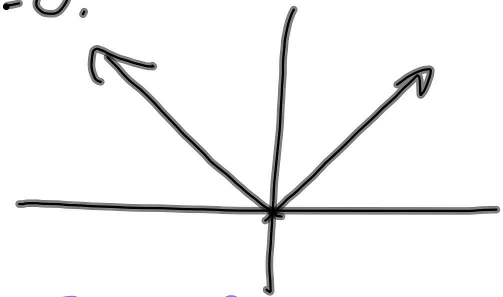
$f(x) = |x|$ doesn't have a derivative at $x = 0$.



$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

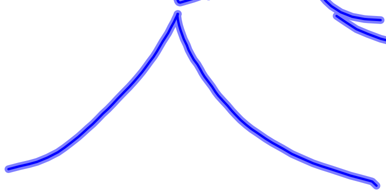
but what happens @ $x = 0$?

$$\lim_{h \to 0^-} \frac{f(0+h) - f}{h} = \frac{-(0+h) - (0)}{h} = \frac{0 - h + 0}{h} = -1$$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \frac{0+h-0}{h} = \frac{h}{h} = 1$$

$$\therefore \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \quad \nexists$$

Not difb$^{\underline{le}}$



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