§1.3

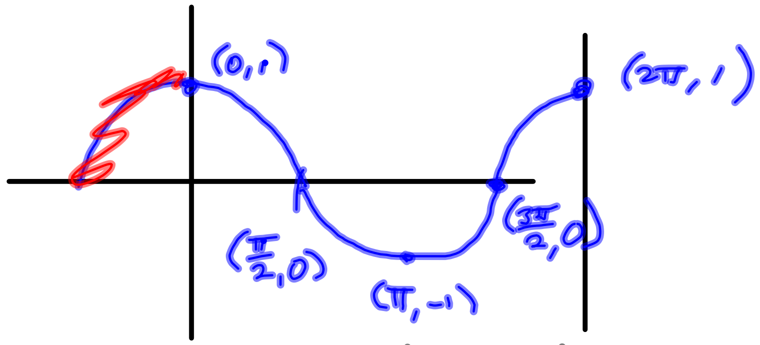
$\frac{1}{2} - \frac{1}{2}\cos x = \frac{1}{2}(1 - \cos x)$

Label key points as ordered pairs. Graph by transformations on a basic function.

Context

See my solutions for an idea of what finished work should look like.

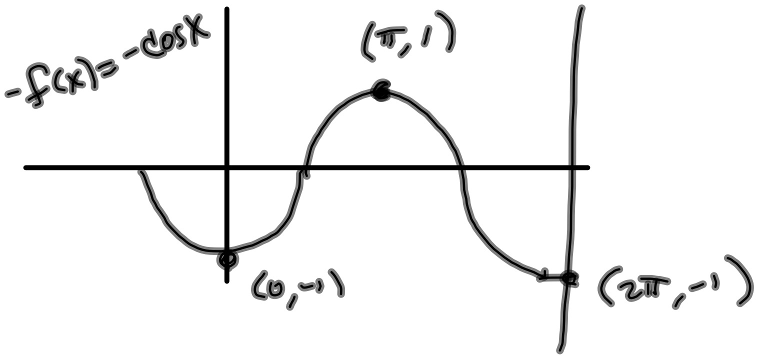
$f(x) = \cos x$



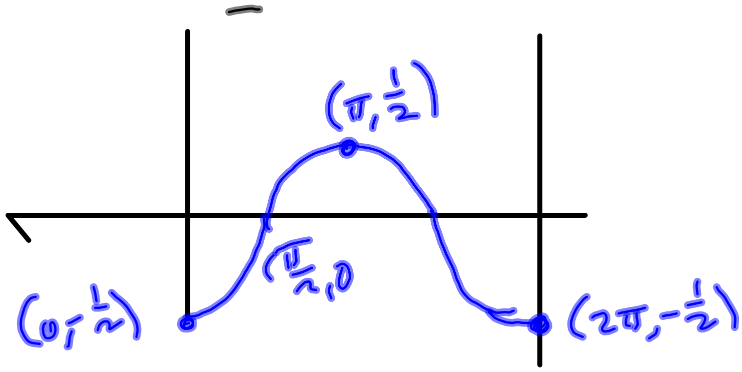
$$-\cos x + 1$$

$$\cos x \to -\cos x \to 1 - \cos x \longrightarrow \frac{1}{2}(1 - \cos x)$$

$$\cos x \to -\cos x \to -\frac{1}{2}\cos x \longrightarrow \frac{1}{2} - \frac{1}{2}\cos x$$

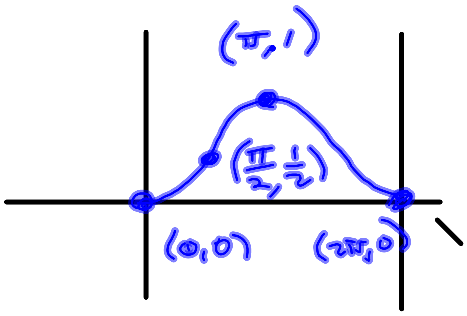


$-\frac{1}{2}\cos x$



$-\frac{1}{2}f(x)+\frac{1}{2}$

$-\frac{1}{2}\cos x+\frac{1}{2}$



$$y = g(x) = 1 - 2x - x^2$$

$$x^2 \longrightarrow -x^2 \longrightarrow -x^2 - 2x + 1$$

Complete the square, instead, to nail the vertex

$$g(x) = -x^2 - 2x + 1$$

$$-g(x) = x^2 + 2x \cancel{-1}$$

$$-g(x) + 1 + 1 = x^2 + 2x + 1^2$$

$$-g(x) + 2 = (x+1)^2$$

$$-g(x) = (x+1)^2 - 2$$

$$g(x) = -(x+1)^2 + 2$$

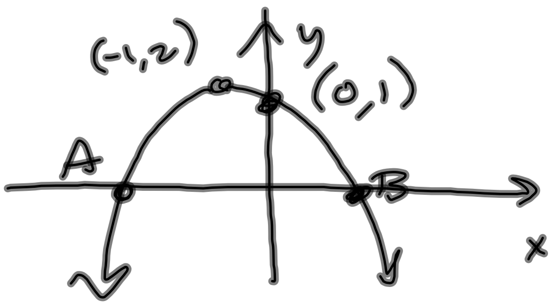
$(h,k) = (-1,2)$

opens down

$$y = m(x - x_1) + y_1$$

$$y = a(x-h) + k$$

$$= a(x-h)^2 + K$$



$$g(x) = -(x+1)^2 + 2 = 0$$

$$-(x+1)^2 = -2$$

$$(x+1)^2 = 2$$

$$x + 1 = \pm\sqrt{2}$$

$$x = -1 \pm \sqrt{2}$$

$$A = (-1-\sqrt{2}, 0)$$

$$B = (-1+\sqrt{2}, 0)$$

Sep 4-12:24 PM

Using the squeeze theorem:

find functions above & below g(x) that approach the same value, squeezing $\lim_{x \to a} g(x)$ in between

Want $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$

$$-1 \le \cos\left(\frac{2}{x}\right) \le +1 \longrightarrow$$

(since $x^4 > 0$ when $x \neq 0$)

$$-x^4 \le x^4 \cos\left(\frac{2}{x}\right) \le x^4$$

$x \to 0$ $\qquad$ $x \to 0$ $\qquad$ $x \to 0$

$$0 \le 0 \le 0$$

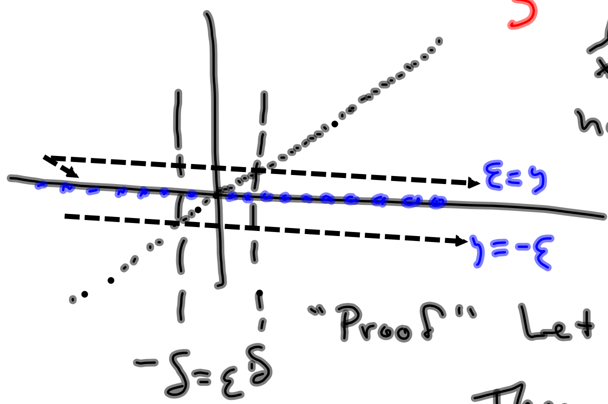
by Squeeze...

§1.8 #64

$$g(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$$

3

$\lim_{x \to c} f(x) = f(c)$ only holds



Claim: It's continuous at $x=0$. ~~tet~~

"Proof" Let $\varepsilon > 0$ be given.

Then $\delta = \varepsilon$ will assure all y-values will stay inside the ε-tube.

Given any $\varepsilon > 0$, x can be made less by taking x close enough to zero. That keeps both $y=0$ & $y=x$ inside the ε-window. That's Formal Continuity.

In general: Everything's continuous on its domain.

Exceptions:

Division by zero

square root of negative

piecewise functions at the suture points.

$$f(x) = \begin{cases} 2x+1 & \text{if } x \leq 1 \\ 3x-7 & \text{if } x > 1 \end{cases}$$

where is it cnt$\underline{s}$?

$(-\infty, 1) \cup (1, \infty)$ for starters

What about $x=1$?

$$\boxed{\lim_{x \to 1} f(x) \overset{?}{=} f(1)} = 2(1)+1 = 3 = \lim_{x \to 1^-} f(x)$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3x-7) = -4 \neq 3 = \lim_{x \to 1^-} f(x)$$

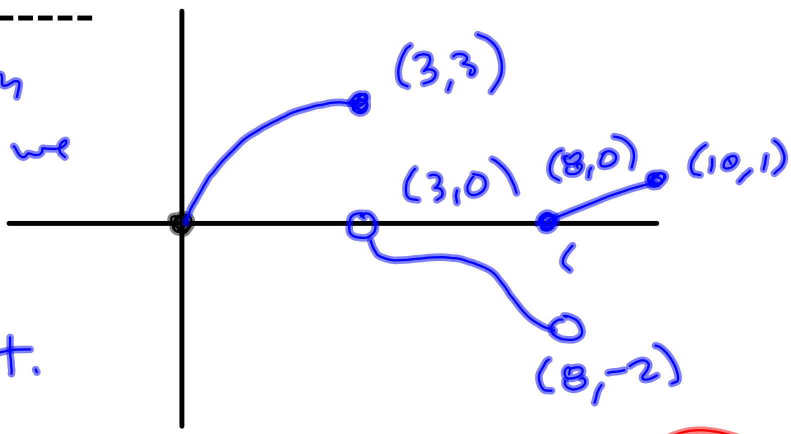
$$\Longrightarrow \lim_{x \to 1} f(x) \nexists$$ NOT Continuous @ $x=1$

Sep 4-12:50 PM

Exception: Left-and right-endpoints of intervals, we speak of continuity from the right and left, respectively.

f(x) is defined on [0, 10]

cnts on [0,3], if we stopped at x=3 It doesn't.



f is cnt^s on [0,3)∪(3,8)∪(8,10]

cnt^s from one side.