§1.7 # 29 was just too easy
Please add #30 to the list of exercises.

$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

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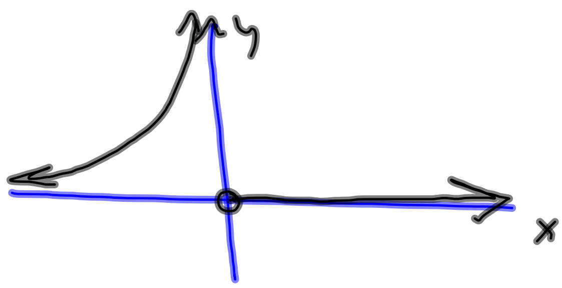
$$f(x) = \frac{1}{x} - \frac{1}{|x|} = \begin{cases} \frac{1}{x} - \frac{1}{x} & \text{if } x > 0 \\ \frac{1}{x} - \frac{1}{-x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} 0 & \text{if } x > 0 \\ \frac{2}{x} & \text{if } x < 0 \end{cases} \implies \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 0 = 0$$



$$|2x^3 - x^2| = |x^2|\,|2x-1|$$

$$= x^2\,|2x-1| = \begin{cases} x^2(2x-1) & \text{if } 2x-1 \geq 0 \\ x^2(-(2x-1)) & \text{if } 2x-1 < 0 \end{cases}$$

$$= \begin{cases} x^2(2x-1) & \text{if } x \geq \frac{1}{2} \\ -x^2(2x-1) & \text{if } x < \frac{1}{2} \end{cases}$$

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§1.7 #29

Claim $\lim_{x \to 2} (x^2-4x+5) = 1$

<u>Scratch</u>:

want $|x^2-4x+5-1| < \varepsilon$

$|x^2-4x+4| < \varepsilon$

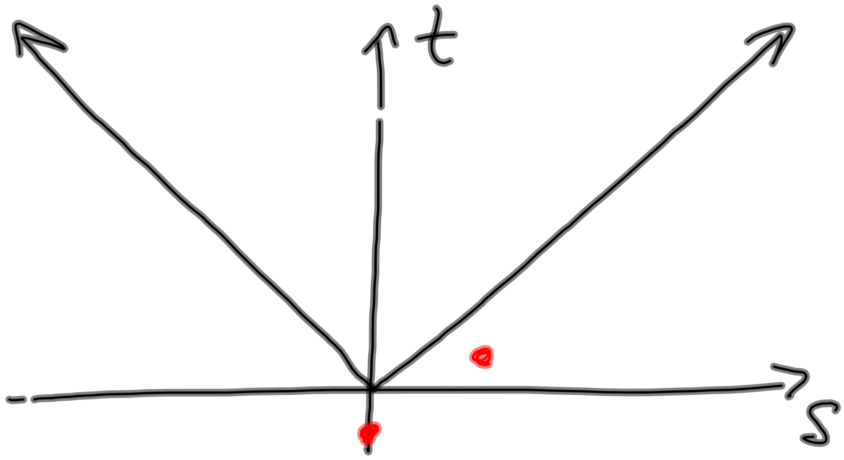
$|x-2|^2 < \varepsilon$

$|x-2|$ will be our $< \delta$ bit

$|x-2| < \sqrt{\varepsilon} = \delta$

<u>Proof</u> Let $\varepsilon > 0$ be given. Define $\delta = \sqrt{\varepsilon}$. Then if $0 < |x-2| < \delta$, we have

$$|x^2-4x+5-1| = |x^2-4x+4| = |x-2|^2 < \delta^2 = (\sqrt{\varepsilon})^2 = \varepsilon \quad \blacksquare$$



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§1.7 #37 Use rationalizing Numerator technique

Claim $\lim_{x \to a} \sqrt{x} = \sqrt{a}$

$(a-b)(a+b) =$

want

$|\sqrt{x} - \sqrt{a}| < \varepsilon$

$$\boxed{|\sqrt{x} - \sqrt{a}| = \left| \frac{\sqrt{x} - \sqrt{a}}{1} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} \right| = \left| \frac{x-a}{\sqrt{x} + \sqrt{a}} \right|}$$

$= \frac{|x-a|}{\sqrt{x} + \sqrt{a}} \overset{\text{want}}{<} \varepsilon$

$\frac{|x-a|}{\sqrt{x} + \sqrt{a}} < \frac{|x-a|}{\sqrt{a}} < \varepsilon$

$\Longrightarrow \underbrace{|x-a|}_{\delta} < \varepsilon \sqrt{a}$

Let $\delta = \sqrt{a}\,\varepsilon$

$\sqrt{x} + \sqrt{a} > \sqrt{a}$

$\frac{1}{\sqrt{x} + \sqrt{a}} < \frac{1}{\sqrt{a}}$

If I can make this small, then

This is automatically even smaller.

§1.7 #37

Claim $\lim_{x \to a} \sqrt{x} = \sqrt{a}$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \varepsilon\sqrt{a}$.

Then if $0 < |x-a| < \delta$, we have

$$|\sqrt{x} - \sqrt{a}| = \frac{|x-a|}{\sqrt{x}+\sqrt{a}} < \frac{|x-a|}{\sqrt{a}} < \frac{\delta}{\sqrt{a}} = \frac{\varepsilon\sqrt{a}}{\sqrt{a}}$$

$$= \varepsilon \ \blacksquare$$

$\leq$ is more precise, since $x=0$ is allowed.

Only things new in §1.8

Definition of continuity

$$\lim_{x \to c} f(x) = f(c)$$

$\sin\left(\frac{1}{x}\right)$ is not continuous @ $x=0$

Major Implications of the definition (which all flow from previous study of limits)

IVT: If f is continuous on $[a,b]$ and L is between $f(a)$ & $f(b)$, then there is a c in (a,b) with $f(c) = L$.

$$\frac{x^2-1}{x-1}$$