Application of this gnarly ε-δ stuff:

§1.2 #11 Area $= 1000\ cm^2 = L$

$f(x) = \pi x^2$, where x = radius of the disk.

(a) what radius produces such a disk?

$f := x \rightarrow \mathrm{Pi}\cdot x^2$

$$x \rightarrow \pi x^2$$

$f(x)$

$$\pi x^2$$

$solve(f(x) = 1000, x)$

$$\frac{10\sqrt{10}}{\sqrt{\pi}}, -\frac{10\sqrt{10}}{\sqrt{\pi}}$$

$\pi x^2 = 1000$

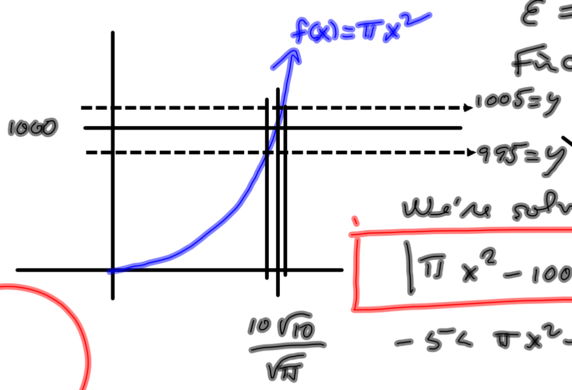
$x^2 = \frac{1000}{\pi}$

$x = \pm\sqrt{\frac{1000}{\pi}} = \pm \frac{10\sqrt{10}}{\sqrt{\pi}}$

(b) what's the tolerance on the radius x to keep the area, $f(x)$, within $\pm 5 cm^2$ of $1000\ cm^2$

$\varepsilon = 5 cm^2$

Find δ!



We're solving

$$\boxed{|\pi x^2 - 1000| < 5}$$

$-5 < \pi x^2 - 1000 < 5$

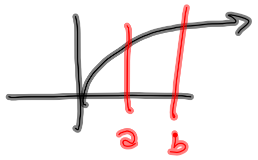
$* \quad 995 < \pi x^2 < 1005$

$\frac{995}{\pi} < x^2 < \frac{1005}{\pi}$

$\sqrt{\frac{995}{\pi}} < \sqrt{x^2} < \sqrt{\frac{1005}{\pi}}$

$a < b$

$\sqrt{a} < \sqrt{b}$



Assume $x > 0 \Rightarrow |x| = x$

$\Rightarrow \sqrt{\frac{995}{\pi}} < x < \sqrt{\frac{1005}{\pi}}$

$\frac{10\sqrt{10}}{\sqrt{\pi}} - \frac{\sqrt{995}}{\sqrt{\pi}} =$

$\sqrt{\frac{1005}{\pi}} - \frac{10\sqrt{10}}{\sqrt{\pi}} =$

$$\frac{10\sqrt{10}}{\sqrt{\pi}} - \operatorname{sqrt}\left(\frac{995}{\mathrm{Pi}}\right)$$

From the left

$$\frac{10\sqrt{10}}{\sqrt{\pi}} - \frac{\sqrt{995}}{\sqrt{\pi}}$$

evalf(%)

$$0.04465900$$

$$\operatorname{sqrt}\left(\frac{1005}{\mathrm{Pi}}\right) - \frac{10\sqrt{10}}{\sqrt{\pi}}$$

From the right

$$\frac{\sqrt{1005}}{\sqrt{\pi}} - \frac{10\sqrt{10}}{\sqrt{\pi}}$$

evalf(%)

$$0.04454749$$

The smaller of the two.

$\boxed{\text{Let } \delta = .04454748}$

Radius must be within $\delta = .04454748$ to stay within the area tolerance (Actually anything smaller would suffice).

(C) In the context of formal definition

of $\lim_{x\to a} f(x) = L$,

$x =$ radius

$L = 1000\ cm^2$ (a fixed area)

$f(x) = \pi x^2$

$a = \frac{10\sqrt{10}}{\sqrt{\pi}}$

$\delta = \frac{\sqrt{1005} - \sqrt{1000}}{\sqrt{\pi}} > .04454748$

We wanted Area between 995 & 1000

$|\pi x^2 - 1000| < 5$

(1) $|f(x) - L| < \varepsilon$

Let $\delta = .04454748 \Longrightarrow$ (1) holds.

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PROVE $\lim_{x\to 3}(2x-1)=5$

Pf Let $\varepsilon>0$ be given. Define $\delta=\frac{\varepsilon}{2}$.

Then, if $0<|x-3|<\delta$, we have

$$|(2x-1)-5| = |2x-6| = 2|x-3| < \delta = 2\cdot\frac{\varepsilon}{2} = \varepsilon \blacksquare$$

PROVE $\lim_{x\to 3}(x^2+3x-1)=17$

Scratch:

$|x^2+3x-1-17|<\varepsilon$

$|x^2+3x-18|<\varepsilon$

$|x-3||x+6|<\varepsilon \longrightarrow |x-3|<\frac{\varepsilon}{|x+6|}$

$|x-3| < \delta$, $|x+6| \to 10$

Need a ceiling (upper bound)

For this, assume $\delta\le 1$

Then $2<x<4$

$2+6<x+6<4+6$

$8<x+6<10$

$\longrightarrow |x+6|<10$

Now we can write the proof.

Let $\varepsilon>0$ be given. Define $\delta=\min\{1,\frac{\varepsilon}{10}\}$.

Then, if $0<|x-3|<\delta$, we have

$$|x^2+3x-1-17| = |x^2+3x-18| = |x-3||x+6|$$

$$<\delta\cdot 10 \le \frac{\varepsilon}{10}\cdot 10 = \varepsilon \blacksquare$$

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