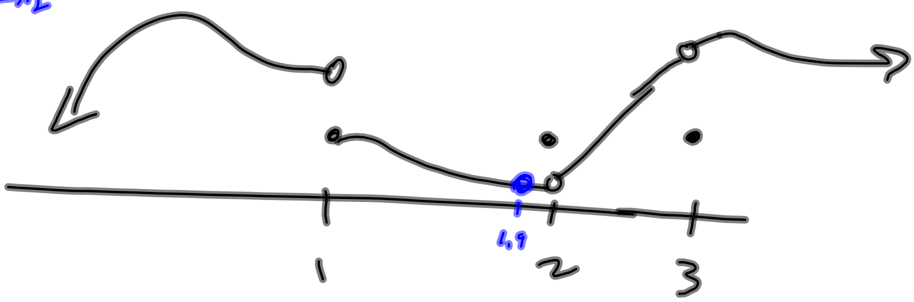
§1.5 #s 11, 27, §

Some of the values are a waste of time

$\lim_{x \to 2} f(x)$, I'd plug in 1.999, 1.9999, 2.001, 2.0001



$\lim_{x \to a} f(x)$ exists on $(-\infty, 1) \cup (1, \infty)$

§1.6 – Main thing is knowing the rules, but you only have to step-by-step on #s 3-9

$\lim_{x \to -2} \sqrt{x^4+3x+6}$ $\sqrt{\lim_{x\to-2}(x^4) + \lim_{x\to-2}(3x) + \lim_{x\to-2}(6)}$

Rules 11, 1,

$= \sqrt{(-2)^4 + 3(-2) + 6}$

Basically, you can move limits inside and plug in the limiting value, except when there's a domain issue, like

$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

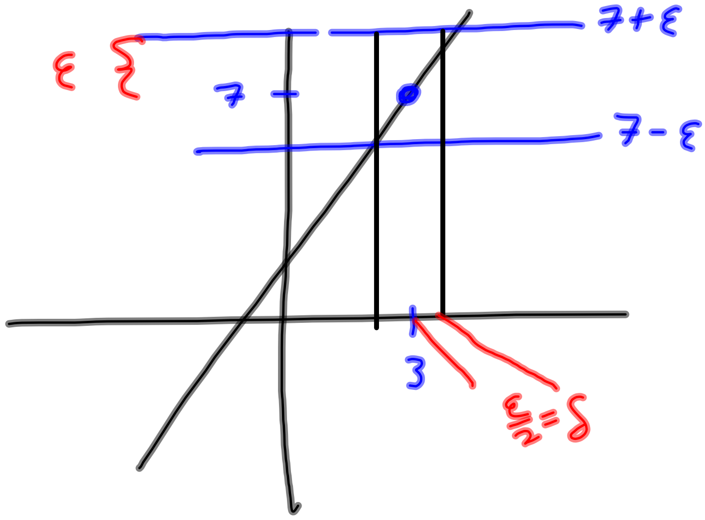
Claim: $\lim_{x \to 3} (2x+1) = 7$

Proof: Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$.

Then if $0 < |x-3| < \delta$, we have

$$|2x+1-7| = |2x-6| = 2|x-3| < 2\delta$$

$$= 2 \cdot \frac{\varepsilon}{2} = \varepsilon \quad \blacksquare$$



Claim $\lim_{x \to 3} (2x+1) = 7$

Scratch : want $|2x+1-7| < \varepsilon$ Finish Line.

Try to isolate an $|x-3|$ for the δ in $\lim_{x \to 3}$

$$|2x-6| < \varepsilon$$
$$|2(x-3)| < \varepsilon$$
$$2|x-3| < \varepsilon$$
$$|x-3| < \frac{\varepsilon}{2} \equiv \delta$$ starting Line.

$|x-3|$ is how far x is from 3.

$|2x+1-7|$ " " " $2x+1$ " " 7.

<u>Claim</u>: $\lim\limits_{x \to 5} (-3x+2) = -13$

<u>Proof</u>: Let $\delta = \frac{\varepsilon}{3}$. If $0 < |x-5| < \delta$,

then $\underbrace{|-3x+2-(-13)|}_{|f(x) - L|} = |-3x+15| = |-3(x-5)|$

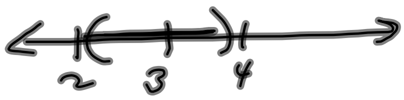
$= 3|x-5| < 3 \cdot \delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ $\blacksquare$

$|-3(x-5)| = |-3|\,|x-5| = 3|x-5|$

$|x-5| < \delta \Longrightarrow$

$\Longleftarrow 3|x-5| < 3\delta$

Let's do a square.



Warm-up. If I know $|x-3|<1$, what's that say about $|x+3|$?

$$|x-3|<1 \Longrightarrow$$

$$-1<x-3<1 \Longrightarrow$$

$$2<x<4 \Longrightarrow$$

$$2+3<x+3<4+3$$

$$5<x+3<7 \rightarrow |x+3|<7, \text{ in particular.}$$

Claim $\lim_{x\to 3} x^2 = 9$

Scratch: Want $|x^2 - 9| < \varepsilon$

$|x+3|\,|x-3| < \varepsilon$

$|x+3|\,\delta$ Assume $\delta < 1$

Then $|x+3| < 7$ $|x-3| \leq 1$

So $|x+3|\,\delta < 7\delta$

That's the scratch. Here's the proof:

Proof Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{7}\}$

Then, if $0 < |x-3| < \delta$, we have

$$|x^2 - 9| = |x+3|\,|x-3| < 7\delta \leq 7\frac{\varepsilon}{7} = \varepsilon \blacksquare$$