1.1's all got $\frac{10}{10}$

1-side only
some context
start fresh page when stuck
Leave Margins / use header.

Last time $\sin\left(\frac{1}{x}\right)$ illustrates how tricky limits can be. $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ $\nexists$

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§1.5 continued

Most of the time small increments will give a numerical result that's good enough.

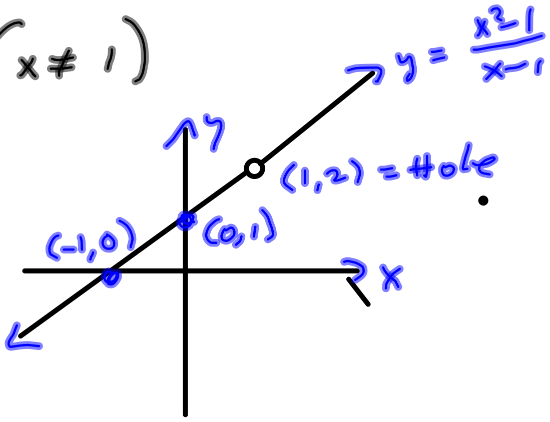
$$\lim_{x\to 1} \frac{x^2-1}{x-1} =$$

Try .999 & 1.001 to get "close."

We want to be exact!

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1} (x+1) = 1+1=2$$

$$\boxed{\frac{x^2-1}{x-1} = x+1} \quad (x \neq 1)$$

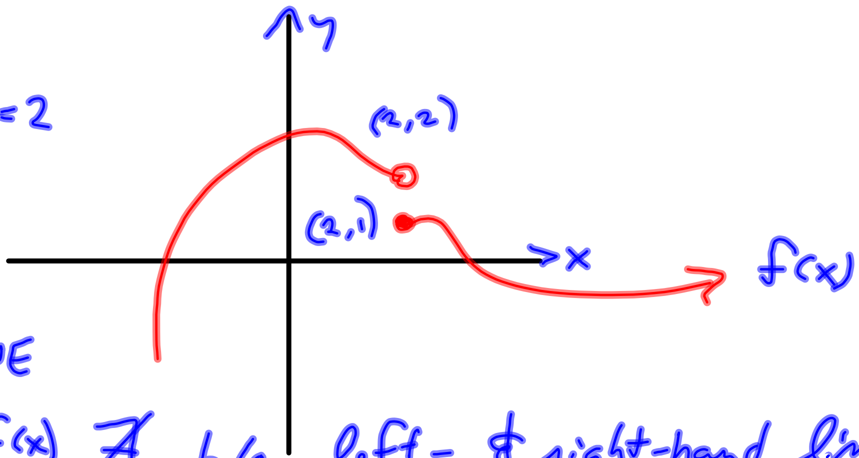


Left-hand limit "Limit as x approaches a from the left of f(x) = L."

$$\lim_{x\to a^-} f(x) = L$$

$$\lim_{x\to 2^-} f(x) = 2$$

$$\lim_{x\to 2^+} f(x) = 1$$



DNE

$\therefore \lim_{x\to 2} f(x) \nexists$, b/c left- & right-hand limits don't agree.

$$f(x)=\begin{cases} 2x+3 & \text{if } x<1 \\ ax^2 & \text{if } x\geq 1 \end{cases}$$

want "a" so the pieces "fit."

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(2x+3) = 2(1)+3 = 5$$

$\Rightarrow$ want $a(1)^2 = 5 = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} ax^2$

$\Rightarrow a=5$

So, $f(x)=\begin{cases} 2x+3 & \text{if } x<1 \\ 5x^2 & \text{if } x\geq 1 \end{cases}$

has a limit @ $x=1$.

§1.6 Limit Laws

1 Limit of the sum is sum of the limits

4 " " " product " product " " "

6 9 " " " power " power " " "

10 11 " " " root " root " " "

3 " " " scalar multiple " scalar mult. " " "

$$\lim_{x\to 3} \frac{x^2+5x}{\sqrt[5]{x}-7} = \frac{\left(\lim\limits_{x\to 3} x\right)^2 + 5^{*}\lim\limits_{x\to 3}(x)}{\sqrt[5]{\lim\limits_{x\to 3}(x)} - 7^{*}}$$

$\lim\limits_{x\to 3} x = 3$, $\lim\limits_{x\to 3} 5 = 5$

$\boxed{\text{\#3-9 in §1.6 is only time you need to get detailed.}}$

Aug 27-12:32 PM

§1.6 #s 1, 2, 5, 9, 10, 11, 13, $\boxed{18B}$, 23, 25, 31,
39, 43, $\boxed{45, 46}$, 47a, 49, Read #54, 58*, 63

Huh?

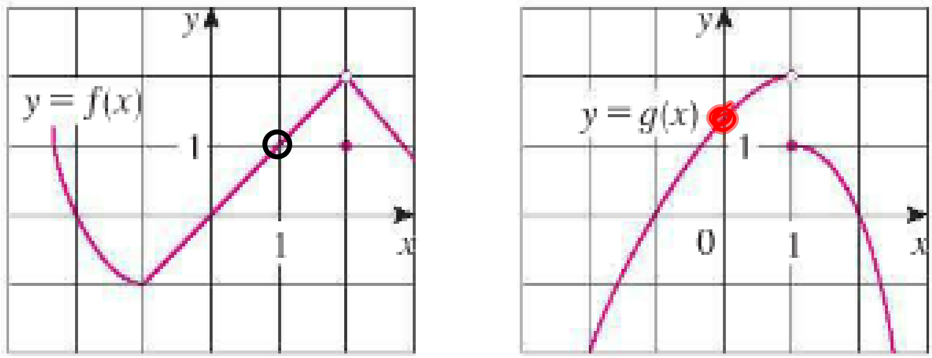
If $f(x)$ is a polynomial, then
$\lim_{x\to a} f(x) = f(a)$.

$R(x) = \frac{f(x)}{g(x)}$ is a $\underbrace{\text{quotient of polynomials}}_{\text{Rational Functions.}}$

then $\lim_{x\to a} R(x) = R(a)$, provided $g(x) \neq 0$.

What can you say about $\lim_{x\to 3} f(x)$ if
you <u>KNOW</u> $\lim_{x\to 3} \frac{f(x)}{x-3} = 5$?

you know $\lim_{x\to 3} f(x) = 0$!



$$\lim_{x \to 0} [f(x)\, g(x)] = \left(\lim_{x \to 0} f(x)\right)\left(\lim_{x \to 0} g(x)\right) = 0 \cdot 1.4 = 0$$

$$\lim_{x \to 1} \sqrt{3 + f(x)} = \sqrt{3 + 1} = \sqrt{4} = 2$$

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Squeeze Theorem

$\lim_{x \to 0} \sin\left(\frac{1}{x}\right) \nexists$

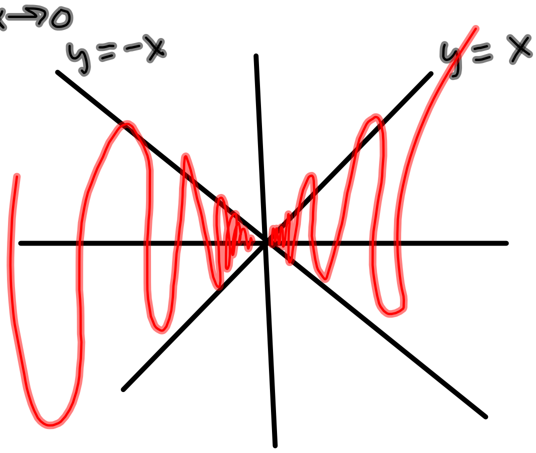
$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

Notice:

$-1 \leq \sin \frac{1}{x} \leq 1$

If $x > 0$, then

$-1x \leq x \sin \frac{1}{x} \leq x$



Then

$\lim_{x \to 0} (-x) \leq \lim_{x \to 0} \left(x \sin\left(\frac{1}{x}\right)\right) \leq \lim_{x \to 0} (x)$

$0 \leq \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) \leq 0$ → Must $= 0$.