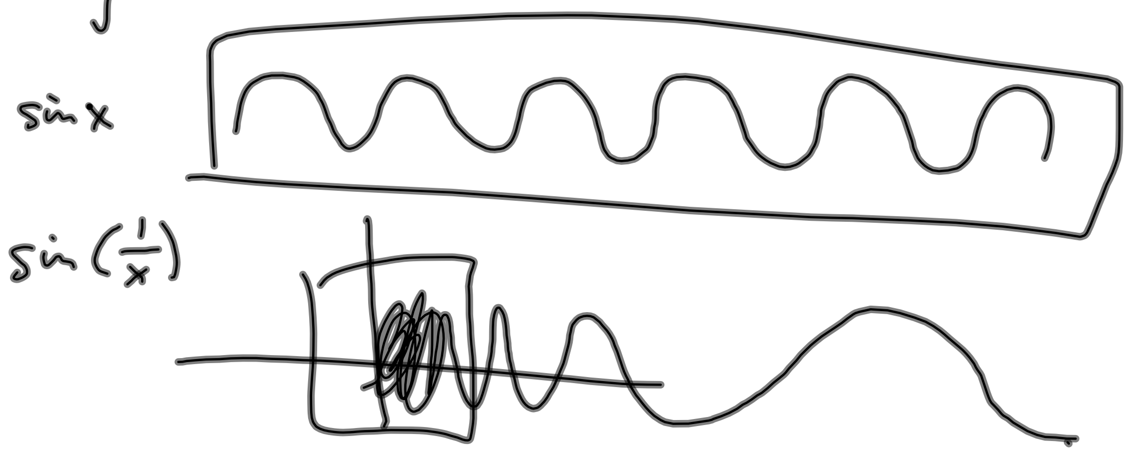
§1.4#9 See Example 4, §1.5

This exercise & this example show that the numerical method can TRICK you on a small number of pathological functions.

$f(x) = \sin\left(\frac{10}{x}\right)$ OR

$f(x) = \sin\left(\frac{\pi}{x}\right)$ oscillate between -1 and $+1$ infinitely often on any neighborhood of $x = 0$



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§1.4#9 $P(1,0)$ is on $\sin\left(\frac{10\pi}{x}\right)$

$Q\left(x, \sin\left(\frac{10\pi}{x}\right)\right)$

Find m_{PQ} for each of these x-values

X	Y_1
2	0
1.5	.86603
1.4	-.4339
1.3	-.823
1.2	.86603
1.1	-.2817
1.001	-.0314

X=1.001

X	Y_1	Y_2
2	0	0
1.5	.86603	1.7321
1.4	-.4339	-1.085
1.3	-.823	-2.743
1.2	.86603	4.3301
1.1	-.2817	-2.817
1.001	-.0314	-31.38

X=2

(x_1,y_1) (x_2,y_2)

$(1,0)$ $Q(2,0)$: $m_{PQ} = \frac{0-0}{2-1} = 0$

$(1,0)$ $(1.5, .86603)$. $m \approx \frac{.86603-0}{1.5-1} \approx 1.7321$

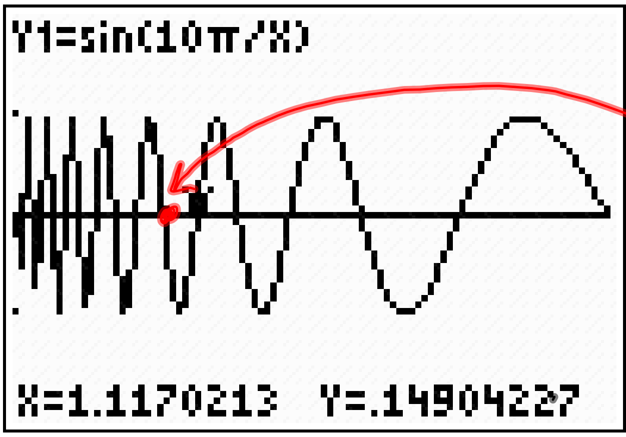
$(1,0)$ $(1.4, -.4339)$ $m = \frac{-.4339-0}{1.4-1} \approx -1.085$

$(1.3, -.823)$ $m = \frac{-.823-0}{1.3-1} \approx -2.743$

$(1.2, .86603)$ $m = \frac{.86603-0}{1.2-1} \approx 4.3301$

$(1.1, -.2817)$ $m = \frac{-.2817-0}{1.1-1} \approx -2.817$

$(1.001, -.0314)$ ≈ -31.38 is probably close.



(1,0) is here

This function IS continuous close to $x=1$. The limit

Does exist.

It Does have slope @ (1,0)

X	Y_2
1	-31.42
1.5	1.7321
1.4	-1.085
1.3	-2.743
1.2	4.3301
1.1	-2.817
1.001	-31.38

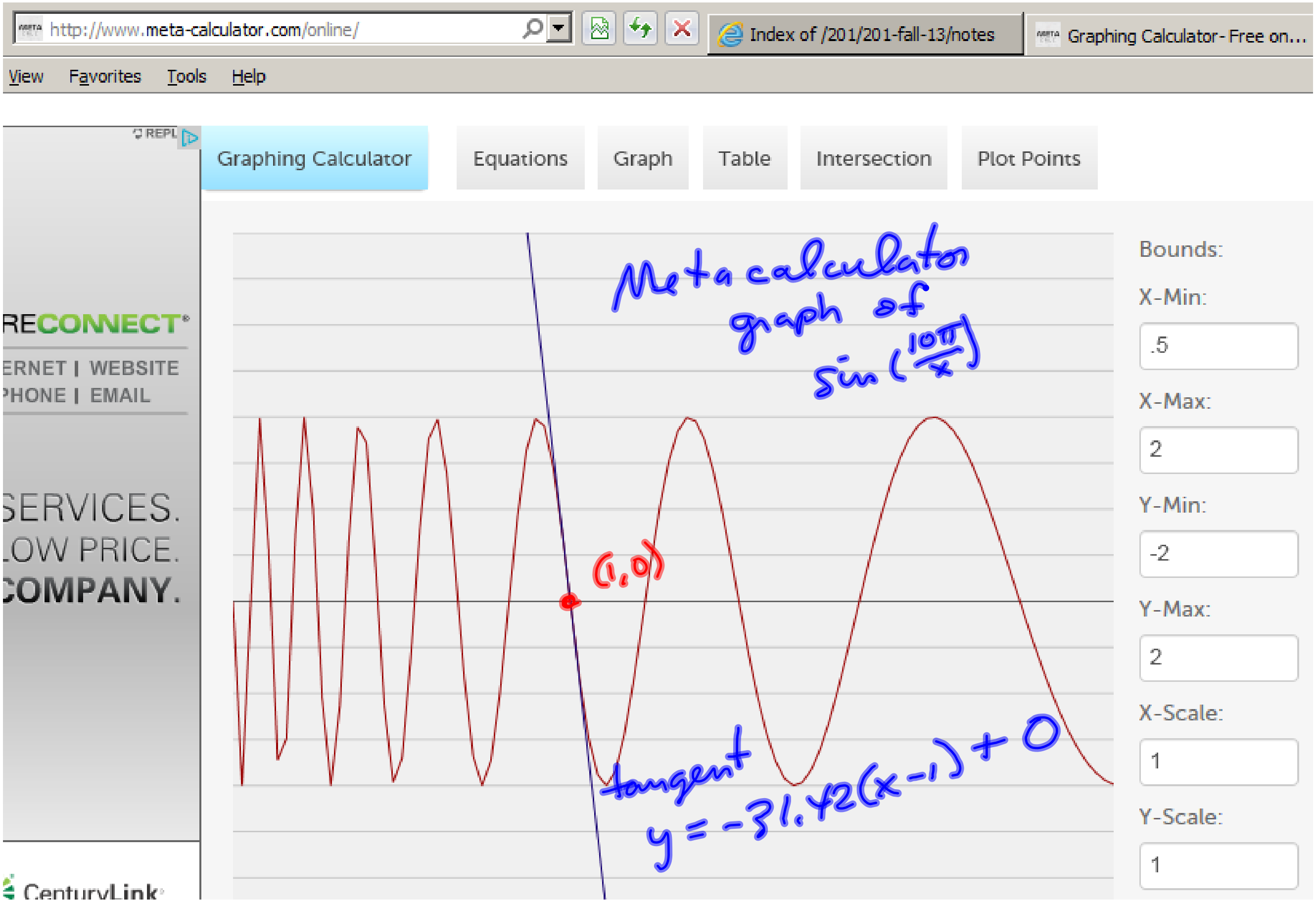
X=1.5

Took x VERY close to $x=1$ & this is a GOOD estimate of the slope @ $x=1$.

Estimated tangent line slope: -31.42

Tangent Line Equation $y = m(x - x_1) + y_1$

$y = -31.42(x-1) + 0$


http://www.meta-calculator.com/online/
Index of /201/201-fall-13/notes
Graphing Calculator - Free on...
View Favorites Tools Help
Graphing Calculator
Equations
Graph
Table
Intersection
Plot Points
Bounds:
X-Min:
.5
X-Max:
2
Y-Min:
-2
Y-Max:
2
X-Scale:
1
Y-Scale:
1
Meta calculator graph of $\sin\left(\frac{10\pi}{x}\right)$
(1,0)
tangent
$y = -31.42(x-1) + 0$

§1.5 The LIMIT of a Function

There are very few functions that won't let you compute the limit by taking x very close to the limiting value.

$\boxed{E4}$ is one of those, and it only causes heartburn @ x=0.

Recall

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$

$$= \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1}$$

$$= \lim_{x\to 1} (x+1) = 2$$

$$f(x) = \sin\left(\frac{\pi}{x}\right)$$

here's how you get tricked:

Want $\lim_{x\to 0} f(x)$

Try $x = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$f(1) = \sin\pi = 0$

$f(\frac{1}{2}) = \sin(2\pi) = 0$

$f(\frac{1}{3}) = \sin(3\pi) = 0$

$f(\frac{1}{4}) = \sin\left(\frac{\pi}{\frac{1}{4}}\right) = \sin(4\pi) = 0$

$f\left(\frac{1}{1000000}\right) = \sin(1000000\pi) = 0$

We got fooled.

Almost never will, except for this one pathological example.