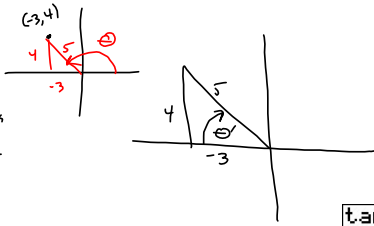


1. We convert $(x,y) = (-3,4)$ to polar coordinates, (r,θ) .

a. Assume $r > 0$ and $\theta \in [0, 2\pi]$. Find the *exact* polar coordinates of the point. To do this, you *should* end up with an arctangent in your answer.

$r = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$
 $\arctan\left(\frac{4}{-3}\right)$ calculator sees this
 θ' is reference angle
 $\arctan\left(-\frac{4}{3}\right) \approx -53.13010235^\circ = -\theta'$
 $\theta = \pi - 53.13010235^\circ$
 $(r, \theta) = (5, \pi + \arctan(-\frac{4}{3}))$
 Details:
 Exact answer
 $\theta = \pi + \arctan(-\frac{4}{3})$




$\tan^{-1}(-4/3)$
 -53.13010235
 $\text{Ans} + 180$
 126.8698976
 $\text{Ans} * \pi / 180$
 2.214297436

b. Approximate your answer in part a, with 4-decimal-place accuracy. Give answer in both radians and degrees.

$$\approx (5, 126.8699^\circ) \approx (5, 2.2143 \text{ rad})$$

c. Find two more representations for your answer in part a.

$(-r, \arctan(-\frac{4}{3}))$, $(-r, \arctan(-\frac{4}{3}) + \pi)$


d. Show the point in a quick sketch.



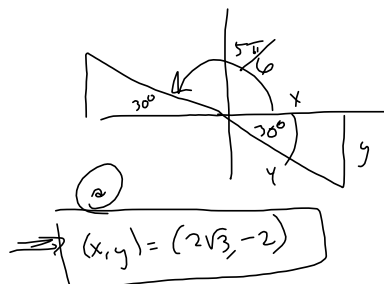
2. Convert $(r, \theta) = \left(-4, \frac{5\pi}{6}\right)$ to rectangular coordinates.

a. Give an exact answer and a decimal answer, accurate to 4 decimal places.

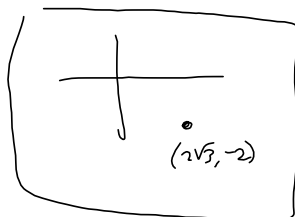
$$5 \sin(-30^\circ) = \frac{y}{4}$$

$$4 \sin(-30^\circ) = y = 4\left(-\frac{1}{2}\right) = -2$$

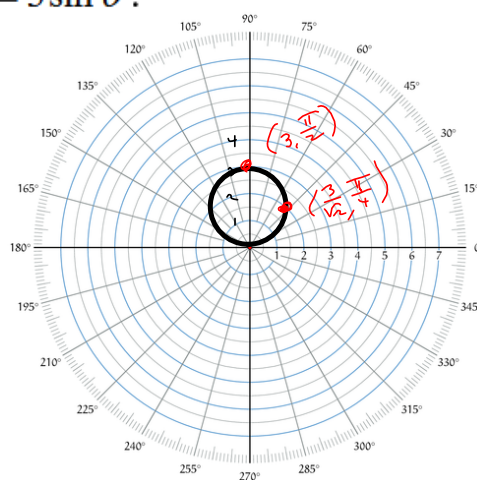
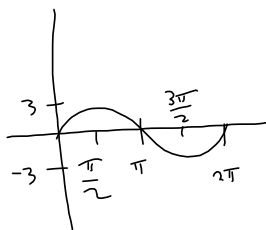
$$x = 4 \cos(-30^\circ) = 4 \frac{\sqrt{3}}{2} = 2\sqrt{3}$$



b. Show the point in a quick sketch.



3. Sketch the graph of $r = 3 \sin \theta$.



θ	r
$\frac{\pi}{4}$	

$$3 \sin \frac{\pi}{4} = \frac{3}{\sqrt{2}}$$

4. Consider the triangle in the figure. Assume lengths are in centimeters.

a. (5 pts) Use the Law of Cosines to find the length of side a.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

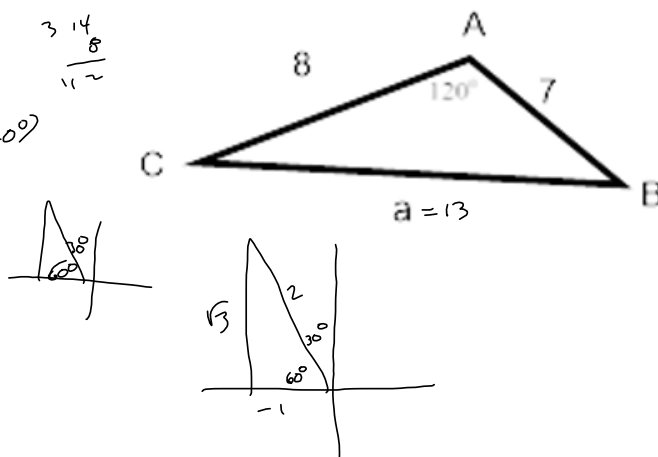
$$= 8^2 + 7^2 - 2(8)(7) \cos(120^\circ)$$

$$= 64 + 49 - 112 \left(-\frac{1}{2}\right)$$

$$= 113 + 56$$

$$= 169 \Rightarrow$$

$$a = \sqrt{169} = 13 = a$$



b. (5 pts) Use the Law of Sines to find angles B and C.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{8} = \frac{\sin(120^\circ)}{13}$$

$$\sin B = \frac{8 \sin(120^\circ)}{13}$$

$$\arcsin(\sin B) =$$

$$\arcsin\left(\frac{8\left(\frac{\sqrt{3}}{2}\right)}{13}\right) = \arcsin\left(\frac{4\sqrt{3}}{13}\right) \approx 32.20422^\circ \approx B$$

$$\text{So } C = 180^\circ - 120^\circ - \arcsin\left(\frac{4\sqrt{3}}{13}\right)$$

$$\approx 27.79578^\circ \approx C$$

B is acute, so $0 < B < 90$, i.e.

$$\arcsin(\sin B) = B$$

If B were in a different quadrant, we'd need to interpret $\sin^{-1}(\sin B)$ from calculator

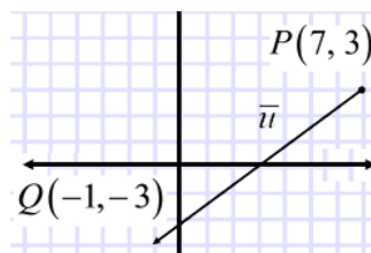
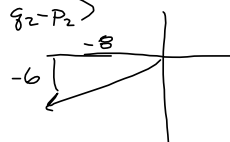
```
126.8698976
Ans*π/180
2.214297436
sin⁻¹(4√(3)/13)
32.2042275
180-120-Ans
27.7957725
```

5. Consider the directed line segment \overrightarrow{PQ} in the figure on the right. I want you to provide some basic facts about the vector \vec{u} :

a. (5 pts) Express the vector $\vec{u} = \overrightarrow{PQ}$ in component form.

Subtract start from end. $\langle q_1 - p_1, q_2 - p_2 \rangle$

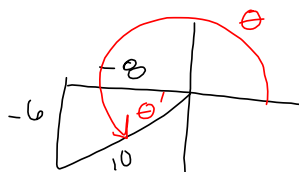
$$\vec{u} = \langle -1 - 7, -3 - 3 \rangle = \langle -8, -6 \rangle = \vec{u}$$



b. (5 pts) Compute the magnitude of \vec{u} . Leave your answer in simplified radical form.

$$\|\vec{u}\| = \sqrt{(-8)^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10 = \|\vec{u}\|$$

c. (5 pts) Find the direction angle of \vec{u} . Use degrees, rounded to 4 places.



$$\arctan\left(\frac{-6}{-8}\right) = \arctan\left(\frac{3}{4}\right)$$

$$\Rightarrow \theta = 180^\circ + \arctan\left(\frac{3}{4}\right)$$

$$\theta \approx 216.8699^\circ$$

$$180 + \tan^{-1}(3/4) \\ 216.8698976$$

6. Let $\vec{u} = \langle 4, 5 \rangle$.

- (5 pts) Express \vec{u} as a linear combination of the canonical (standard) unit vectors \vec{i} and \vec{j} .
- (5 pts) What's another word for the sum of 2 vectors?

(a) $\vec{u} = 4\vec{i} + 5\vec{j}$

(b) Resultant.

7. Forces with magnitudes $\|\vec{u}\| = 90 \text{ N}$ and $\|\vec{v}\| = 25\sqrt{2} \text{ N}$ are acting on a hook, as shown in the figure.
- (5 pts) Express \vec{u} and \vec{v} in component form.
 - (5 pts) Express the resultant force, in component form.
 - (5 pts) Find the direction angle of the resultant force, in degrees, rounded to 4 decimal places.

$$\textcircled{a} \quad y = r \sin \theta$$

$$x = r \cos \theta$$

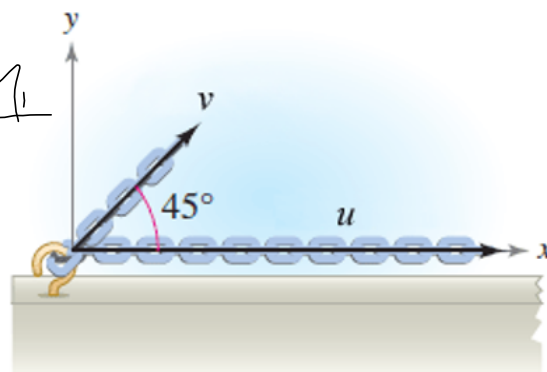
$$\vec{u} = \langle 90 \cos 0^\circ, 90 \sin 0^\circ \rangle$$

$$= \langle 90, 0 \rangle = \vec{u}$$

$$\vec{v} = \langle 25\sqrt{2} \cos 45^\circ, 25\sqrt{2} \sin 45^\circ \rangle$$

$$= \langle (25\sqrt{2})\left(\frac{1}{\sqrt{2}}\right), 25\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) \rangle$$

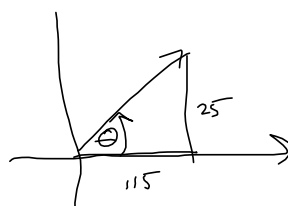
$$= \langle 25, 25 \rangle = \vec{v}$$



$$\textcircled{b} \quad \vec{u} + \vec{v} = \langle 90, 0 \rangle + \langle 25, 25 \rangle$$

$$= \langle 90 + 25, 0 + 25 \rangle$$

$$= \langle 115, 25 \rangle = \vec{u} + \vec{v}$$



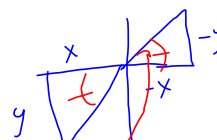
$$\textcircled{c} \quad \theta = \arctan\left(\frac{25}{115}\right) = \arctan\left(\frac{5}{23}\right) = \theta \approx 12.2648^\circ$$

arctangent gave a correct answer, with worries about interpretation. It would require interpretation if we were in QII or QIII

QII $\arctan\left(\frac{y}{x}\right) = \arctan(\text{negative})$
 $\theta = \pi + \arctan\left(\frac{y}{x}\right)$
 arctan sees it here in QIV



QIII $\arctan\left(\frac{y}{x}\right) = \arctan(\text{positive})$, so arctangent "thinks" we're in QI.
 So $\pi + \arctan\left(\frac{y}{x}\right)$ will give us θ



8. Let $f(x) = 3x^3 - 8x^2 + 10x - 4$.

a. (5 pts) Use synthetic division to find $f(2)$.

Remainder Theorem.

Divide by $x-2$

$$\begin{array}{r|rrrr} 2 & 3 & -8 & 10 & -4 \\ & & 6 & -4 & 12 \\ \hline & 3 & -2 & 6 & 8 \end{array}$$

$8 = f(2)$

extra
Interpretation:

$$f(x) = (x-2)(3x^2 - 2x + 6) + 8$$

b. (5 pts) Use synthetic division to show that $x = 1+i$ is a solution of the equation $f(x) = 0$.

$$\begin{array}{r|rrrr} 1+i & 3 & -8 & 10 & -4 \\ & & 3+3i & -8-2i & 4 \\ \hline & 3 & -5+3i & 2-2i & 0 \end{array}$$

See? $1+i$ is a zero of $f(x)$!

$$\begin{aligned} 3(1+i) &= 3+3i \\ (-5+3i)(1+i) &= -5-5i+3i+3i^2 \\ &= -5-2i-3 = -8-2i \end{aligned}$$

$$\begin{aligned} (2-2i)(1+i) &= 2(1-i)(1+i) \\ &= 2(1+i^2) = 4 \\ (a+bi)(a-bi) &= a^2+b^2 \end{aligned}$$

So $f(x) = (x - (1+i))(3x^2 + (-5+3i)x + (2-2i))$
↳ depressed polynomial

c. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

$$\begin{array}{r|rrrr} 1-i & 3 & -5+3i & 2-2i \\ & & 3-3i & -2+2i \\ \hline & 3 & -2 & 0 \end{array}$$

Sweet!

$$(x - (1+i))(x - (1-i))(3x - 2) = f(x)$$

Split into linear factors

(x in all factors)

Conjugate Pairs Theorem says
if $a+bi$ is a root,
then so is $a-bi$, if $f(x)$
has real coefficients.
Dividing by $(x - (1-i))$

9. Let $z = 8 - 8i \Rightarrow \bar{z} = 8 + 8i$

a. (5 pts) Find $z + \bar{z}$ and $z\bar{z}$, where \bar{z} is the complex conjugate of z .

$$z + \bar{z} = 8 - 8i + (8 + 8i) = 16 = z + \bar{z}$$

$$z\bar{z} = (8 - 8i)(8 + 8i) = 8^2 + 8^2 = 128 = z\bar{z}$$

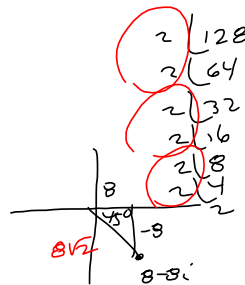
b. (5 pts) Express z in trigonometric form.

$$|z| = \sqrt{a^2 + b^2} = \sqrt{z\bar{z}} = \sqrt{128} = 8\sqrt{2}$$

$$\theta = -\frac{\pi}{4}$$

$$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$z = 8\sqrt{2} \left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right) \right)$$



10. Let $z = 16 \left(\cos \left(\frac{5\pi}{3} \right) + i \sin \left(\frac{5\pi}{3} \right) \right)$. $\left(\frac{5\pi}{3} \right) \left(\frac{180}{\pi} \right) = (5)(60)^\circ = 300^\circ$

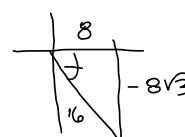


a. (5 pts) Express z in standard form.

Re: $x = 16 \cos \frac{5\pi}{3} = 16 \left(\frac{1}{2} \right) = 8$

Im: $y = 16 \sin \frac{5\pi}{3} = 16 \left(-\frac{\sqrt{3}}{2} \right) = -8\sqrt{3}$

$z = 8 - 8\sqrt{3}i$



b. (5 pts) Find the principal 4th root of z , i.e., find $\sqrt[4]{z}$. Leave z in trigonometric form for this.

$\sqrt[4]{z} = \sqrt[4]{16} \left(\cos \left(\frac{5\pi}{12} \right) + i \sin \left(\frac{5\pi}{12} \right) \right) = \sqrt[4]{2} \left(\cos \left(\frac{5\pi}{12} \right) + i \sin \left(\frac{5\pi}{12} \right) \right) = \sqrt[4]{z}$

$3 \cdot 4 = 12$

c. (5 pts) Now, find *all* the 4th roots of z , in trigonometric form.

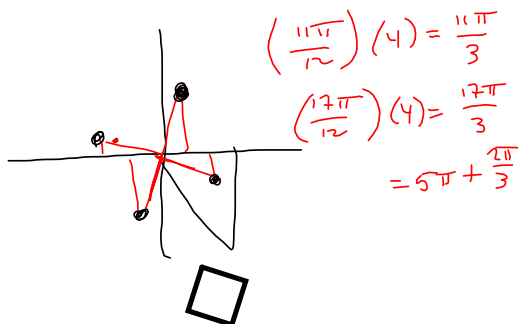
$\frac{2\pi}{4} = \frac{\pi}{2} = \frac{6\pi}{12}$ $\frac{5\pi + 6\pi}{12} = \frac{11\pi}{12}$, $\frac{11\pi + 6\pi}{12} = \frac{17\pi}{12}$, $\frac{17\pi + 6\pi}{12} = \frac{23\pi}{12}$

$\sqrt[4]{z} = \text{ans to b.}$

$2 \left(\cos \left(\frac{11\pi}{12} \right) + i \sin \left(\frac{11\pi}{12} \right) \right)$

$2 \left(\cos \left(\frac{17\pi}{12} \right) + i \sin \left(\frac{17\pi}{12} \right) \right)$

$2 \left(\cos \left(\frac{23\pi}{12} \right) + i \sin \left(\frac{23\pi}{12} \right) \right)$



d. (5 pts) Find the trigonometric form of z^2 .

$16^2 \left(\cos \left(\frac{10\pi}{3} \right) + i \sin \left(\frac{10\pi}{3} \right) \right)$

e. (5 pts) Finally, let $w = 3 \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right)$, and find the trigonometric form of the product $z \cdot w$.

$zw = 16 \left(\cos \left(\frac{5\pi}{3} \right) + i \sin \left(\frac{5\pi}{3} \right) \right) \cdot 3 \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right)$

$= 48 \left(\cos \left(\frac{23\pi}{12} \right) + i \sin \left(\frac{23\pi}{12} \right) \right) = zw$

$\frac{5\pi}{3} + \frac{\pi}{4}$
 $= \frac{5\pi}{3} \cdot \frac{4}{4} + \frac{\pi}{4} \cdot \frac{3}{3}$
 $= \frac{20\pi + 3\pi}{12}$

B1 (5 pts) Find the area of the triangle in the 1st problem.

M1 Heron's

$$s = \frac{a+b+c}{2} = \frac{13+8+7}{2} = 14$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{14(14-13)(14-8)(14-7)}$$

$$= \sqrt{14(1)(6)(7)}$$

$$= \sqrt{588}$$

$$= 14\sqrt{3}$$

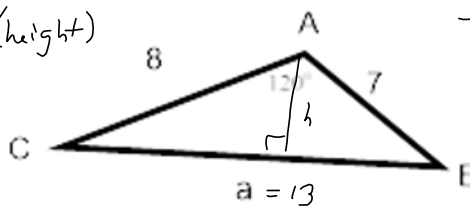
$$\begin{array}{r} 2 \overline{) 588} \\ \underline{294} \\ 3 \overline{) 147} \\ \underline{147} \\ 7 \end{array}$$

$$\sin^{-1}\left(\frac{7\sqrt{3}}{26}\right)$$

$$27.7957725$$

M2 $A = \frac{1}{2}(\text{base})(\text{height})$

$$\begin{array}{r} 42 \\ 14 \\ \hline 168 \\ 420 \\ \hline 588 \end{array}$$



$$\frac{h}{8} = \sin C$$

$$h = 8 \sin C$$

$$h = 8 \left(\frac{7\sqrt{3}}{26} \right)$$

$$= \frac{28\sqrt{3}}{13}$$

$$A = \frac{1}{2}(\text{base})(\text{height})$$

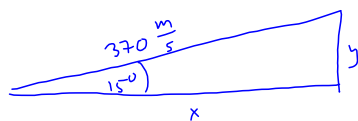
$$= \frac{1}{2}(13)\left(\frac{28\sqrt{3}}{13}\right) = 14\sqrt{3} = A$$

$$\frac{\sin C}{7} = \frac{\sin A}{13}$$

$$\sin C = \frac{7 \frac{\sqrt{3}}{2}}{13}$$

B2 A gun with a muzzle velocity of 370 meters per second is fired, with an angle of 15° from the horizontal.

- a. (5 pts) Find the horizontal and vertical components of the bullet, as it leaves the muzzle, accurate to 4 decimal places.



$$x = 370 \cos 15^\circ \approx 357.3925557 \text{ m/s} = \text{Horizontal component}$$

$$y = 370 \sin 15^\circ \approx 95.76304669 \text{ m/s} = \text{Vertical component}$$

$$\begin{aligned} 370 \cos(15) &= 357.3925557 \\ 370 \sin(15) &= 95.76304669 \end{aligned}$$

- b. (5 pts) Use a half-angle formula to find the *exact* value for the answer to the previous.

$\cos 15^\circ = \sqrt{\frac{1 + \cos 30^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

$\sin 15^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

so, $x = (370) \left(\frac{\sqrt{2 + \sqrt{3}}}{2} \right) = 185 \sqrt{2 + \sqrt{3}} \text{ m/s}$

$y = (370) \left(\frac{\sqrt{2 - \sqrt{3}}}{2} \right) = 185 \sqrt{2 - \sqrt{3}} \text{ m/s}$

These are components of the velocity!

- c. (5 pts) Using $-9.8 \frac{m}{s^2}$ for the acceleration due to gravity, and neglecting air

friction, predict where and when the bullet will hit the ground, in the gun question.

Find when $y=0$ & then find x .

$$x = v_x t \text{ retr.}, \text{ so } \langle x, y \rangle = \langle 185 \sqrt{2 + \sqrt{3}} t, 185 \sqrt{2 - \sqrt{3}} t \rangle$$

Fall in body physics

$$\text{height} = \frac{1}{2} g t^2 + v_0 t + h_0 \quad \text{Newton's Apple.}$$

$$\begin{aligned} &= \frac{1}{2} (-9.8) t^2 + 185 \sqrt{2 - \sqrt{3}} t \\ &= -4.9 t^2 + 185 \sqrt{2 - \sqrt{3}} t \end{aligned}$$

$$\begin{aligned} \frac{1}{2} g t^2 + v_0 t &\stackrel{\text{SET}}{=} 0 \\ t \left(\frac{1}{2} g t + v_0 \right) &= 0 \\ t=0 \text{ or } -4.9 t + v_0 &= 0 \\ v_0 &= 4.9 t \end{aligned}$$

times

$$\text{So } x = v_x t$$

$$\approx (185 \sqrt{2 + \sqrt{3}}) (19.54347892)$$

$$\approx 6984.693878 \text{ m}$$

Down Range

$$\begin{aligned} \text{Ans} &= 185 \sqrt{2 + \sqrt{3}} \\ &= 6984.693878 \\ &= 19.5435 \times 357.3925 \\ &= 6984.701412 \end{aligned}$$

$$t \approx 19.54347892 \approx \frac{185 \sqrt{2 - \sqrt{3}}}{4.9} = \frac{v_0}{4.9} = t$$

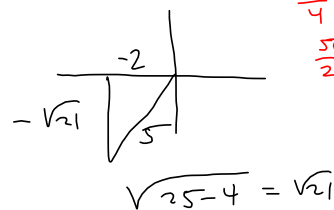
$$\begin{aligned} 185 \sqrt{2 - \sqrt{3}} / 4.9 \\ 19.54347892 \end{aligned}$$

$$357.3925557$$

Left Bottom

B3 (5 pts) Find $\sin(2u)$, $\cos(2u)$ and $\tan(2u)$, given that $\cos(u) = -\frac{2}{5}$ and $\sin(u) < 0$. Use the 1st two answers to *build* the 3rd. It's *silly* to go back to your cheat sheet and deal with the mess.

$$\begin{aligned}\sin(2u) &= 2\sin u \cos u \\ &= 2\left(-\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right) \\ &= 2\left(\frac{-\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right) \\ &= \frac{4\sqrt{21}}{25} = \sin(2u)\end{aligned}$$



$$\begin{aligned}\frac{5\pi}{4} &< u < \frac{3\pi}{2} \\ \frac{5\pi}{2} &< 2u < 3\pi\end{aligned}$$

$$\cos(2u) = 2\cos^2 u - 1 = 2\left(-\frac{2}{5}\right)^2 - 1 = \frac{8}{25} - \frac{25}{25} = \left(-\frac{17}{25} = \cos(2u)\right)$$

$$\tan(2u) = \frac{\sin(2u)}{\cos(2u)} = \left(\frac{4\sqrt{21}}{25}\right)\left(-\frac{25}{17}\right) = \left(-\frac{4\sqrt{21}}{17} = \tan(2u)\right)$$

B4 (5 pts) Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = -\frac{2}{5}$ and $\sin(u) < 0$. Use the 1st two answers to *build* the 3rd. It's *silly* to go back to your cheat sheet and deal with the mess.

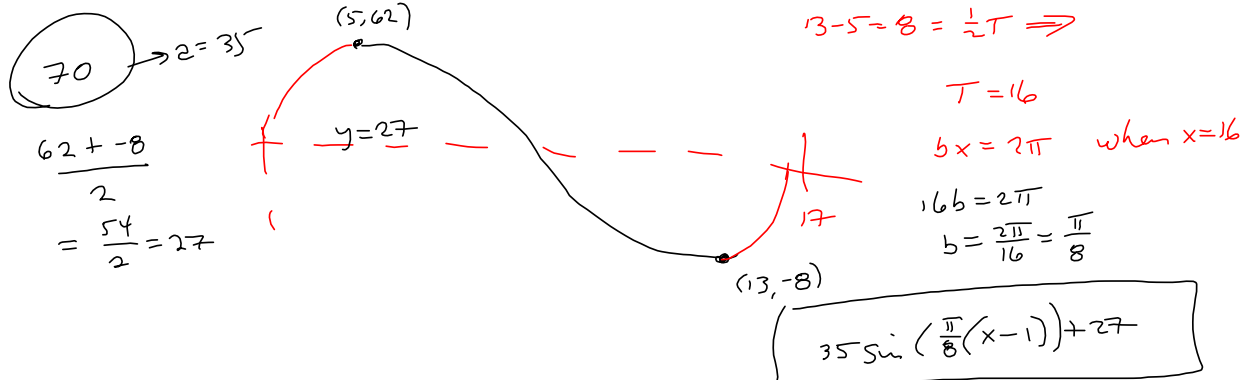
$$\begin{aligned} \sin\left(\frac{u}{2}\right) &= \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - (-\frac{2}{5})}{2}} = \sqrt{\frac{\frac{7}{5}}{\frac{10}{5}}} = \sqrt{\frac{7}{10}} \\ &= \frac{\sqrt{7}}{\sqrt{10}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{10} = \sin\left(\frac{u}{2}\right) \end{aligned}$$

$\frac{5\pi}{4} < u < \frac{3\pi}{2}$
 $\frac{5\pi}{8} < \frac{u}{2} < \frac{3\pi}{4}$ QII
 $\sin\left(\frac{u}{2}\right) > 0$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 - \frac{2}{5}}{2}} = -\sqrt{\frac{\frac{3}{5}}{\frac{10}{5}}} = -\sqrt{\frac{3}{10}} = -\frac{\sqrt{3}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{\sqrt{30}}{10} = \cos\left(\frac{u}{2}\right)$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\frac{\sqrt{14}}{10}}{-\frac{\sqrt{30}}{10}} = -\frac{\sqrt{14}}{\sqrt{30}} = -\sqrt{\frac{14}{30}} = -\sqrt{\frac{7}{15}} = -\frac{\sqrt{7}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = -\frac{\sqrt{105}}{15} = \tan\left(\frac{u}{2}\right)$$

B5 (5 pts) Build a sine function that achieves its maximum height of $y = 62$ meters at time $x = 5$ seconds and its minimum height of $y = -8$ meters at $x = 13$ seconds.



B6 (5 pts) Find all solutions of the equation $2\sin^2(3x) - 1 = 0$ in the interval $[0, 2\pi)$.

$$2u^2 - 1 = 0$$

$$2u^2 = 1$$

$$u^2 = \frac{1}{2}$$

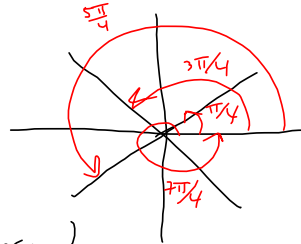
$$u = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

$$0 \leq x < 2\pi$$

$$0 \leq 3x < 6\pi$$

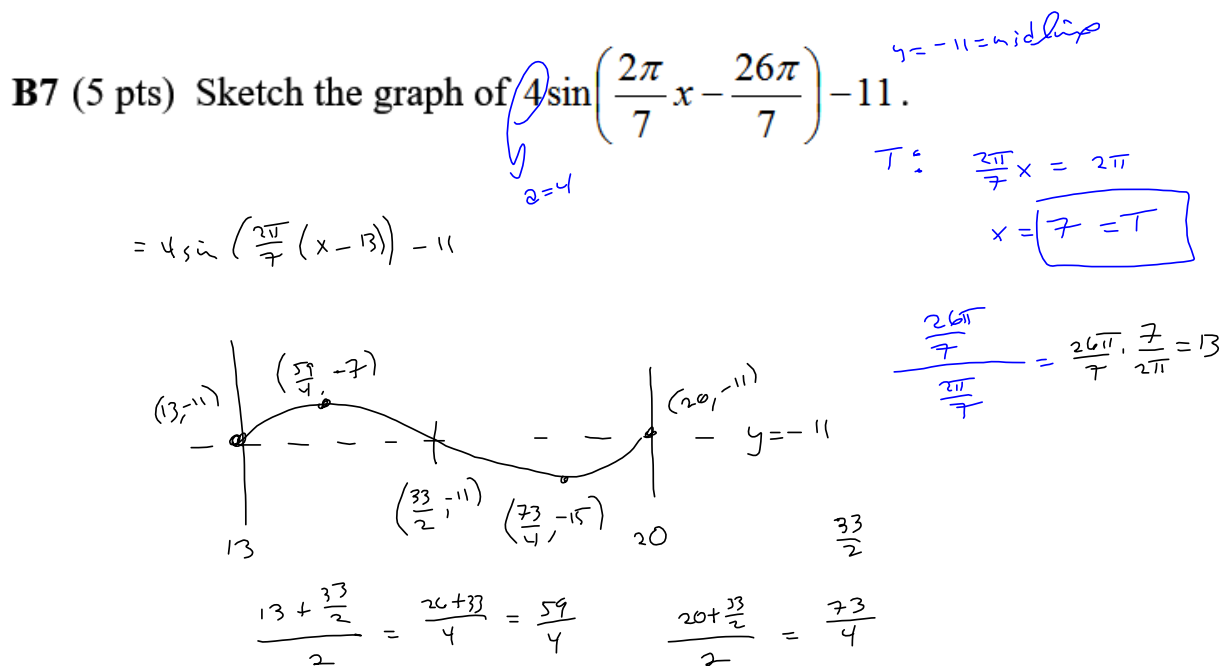
looking for all $3x \in [0, 6\pi)$ that solve the eqn!

3x around circle!



$$3x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{19\pi}{4}, \frac{21\pi}{4}, \frac{23\pi}{4}$$

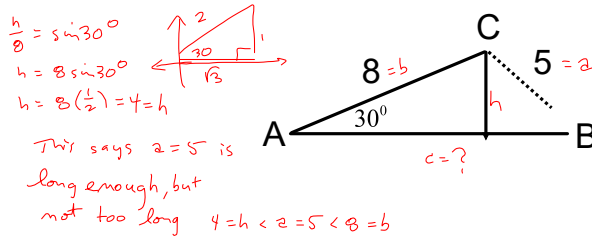
$$\Rightarrow x = \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$$



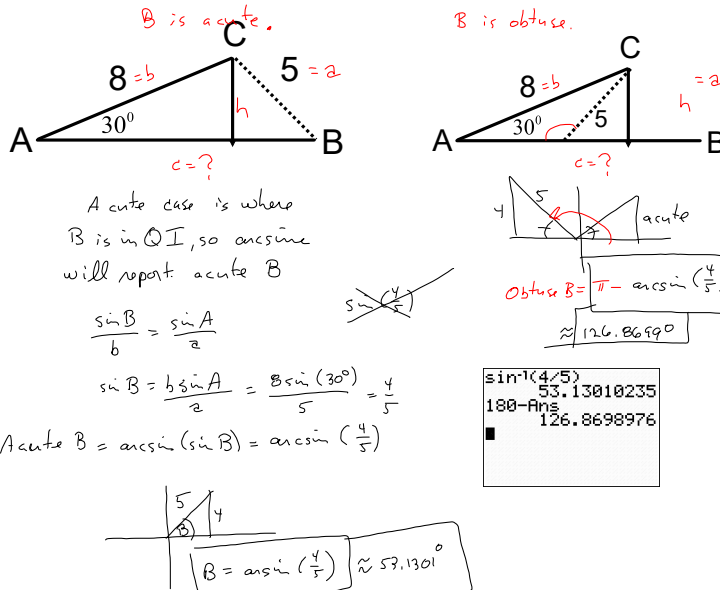
B8 The triangle described has 2 possible solutions: (See Figure on Right)

Angle $A = 30^\circ$, side $b = 8$ and side $a = 5$.

- a. (5 pts) Prove there are 2 possible triangles from this ambiguous information.



- b. (5 pts) Find both triangles.



- c. (5 pts) Use your work to find the area of both triangles.

We already found h .

$$A = \frac{1}{2} b h = \frac{1}{2} c \cdot 4 = 2c \approx 2 \cdot 9.92820323 \approx 19.85640646 \approx \text{AREA}$$

Acute: Find c :

$$C = 180^\circ - B - A$$

$$= 180^\circ - \arcsin(\frac{4}{5}) - 30^\circ$$

$$\approx 96.8698976^\circ$$

$\sin^{-1}(4/5)$
 53.13010235
 $180 - \text{Ans}$
 126.8698976
 $10 * \sin(\text{Ans})$
 9.92820323

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} = \frac{5 \sin(96.8698976^\circ)}{\sin(30^\circ)} = 10 \sin(96.8698976^\circ)$$

$$\approx 9.92820323$$

Obtuse case:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} = \frac{5 \sin(\arcsin(\frac{4}{5}) - 30^\circ)}{\sin(30^\circ)}$$

$\sin^{-1}(4/5)$
 53.13010235
 $180 - \text{Ans}$
 126.8698976
 $10 * \sin(\text{Ans})$
 9.92820323

$\sin^{-1}(4/5)$
 53.13010235
 $180 - \text{Ans}$
 126.8698976
 $10 * \sin(\text{Ans})$
 9.92820323

$\frac{1}{2} b h$
 $= \frac{1}{2} c \cdot 4 = 2c \approx 19.85640646 \approx \text{Area}$

