MAT 122 Test 4 – Fall, 2016 Name_____
100 Points NO GRAPHING CALCULATORS!!!

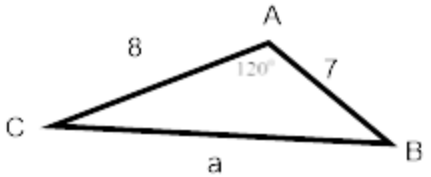
I think you know the drill on margins and legibility. I can't give points for what I can't read. Take a minute, at the end, to make sure your work is organized and submitted in proper order.

NEW MATERIAL from Chapter 6:

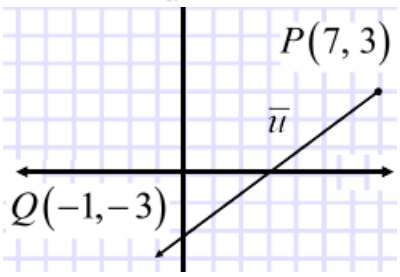
1. We convert $(x, y) = (-3, 4)$ to polar coordinates, (r, θ).
   a. Assume $r > 0$ and $\theta \in [0, 2\pi]$. Find the *exact* polar coordinates of the point. To do this, you *should* end up with an arctangent in your answer.
   b. Approximate your answer in part a, with 4-decimal-place accuracy. Give answer in both radians and degrees.
   c. Find two more representations for your answer in part a.
   d. Show the point in a quick sketch.
2. Convert $(r, \theta) = \left(-4, \frac{5\pi}{6}\right)$ to rectangular coordinates.
   a. Give an exact answer and a decimal answer, accurate to 4 decimal places.
   b. Show the point in a quick sketch.
3. Sketch the graph of $r = 3\sin\theta$.

OLD MATERIAL

4. Consider the triangle in the figure. Assume lengths are in centimeters.
   a. (5 pts) Use the Law of Cosines to find the length of side a.
   b. (5 pts) Use the Law of Sines to find angles B and C.

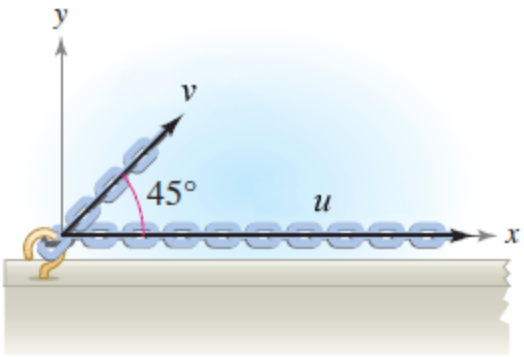


5. Consider the directed line segment $\overrightarrow{PQ}$ in the figure on the right. I want you to provide some basic facts about the vector $\bar{u}$:
   a. (5 pts) Express the vector $\bar{u} = \overrightarrow{PQ}$ in component form.
   b. (5 pts) Compute the magnitude of $\bar{u}$. Leave your answer in simplified radical form.
   c. (5 pts) Find the direction angle of $\bar{u}$. Use degrees, rounded to 4 places.



6. Let $\bar{u} = \langle 4, 5 \rangle$.
   a. (5 pts) Express $\bar{u}$ as a linear combination of the canonical (standard) unit vectors $\bar{i}$ and $\bar{j}$.
   b. (5 pts) What's another word for the sum of 2 vectors?

7. Forces with magnitudes $\|\bar{u}\| = 90$ **N** and $\|\bar{v}\| = 25\sqrt{2}$ **N** are acting on a hook, as shown in the figure.



   a. (5 pts) Express $\bar{u}$ and $\bar{v}$ in component form.
   b. (5 pts) Express the resultant force, in component form.
   c. (5 pts) Find the direction angle of the resultant force, in degrees, rounded to 4 decimal places.

8. Let $f(x) = 3x^3 - 8x^2 + 10x - 4$.
   a. (5 pts) Use synthetic division to find $f(2)$.
   b. (5 pts) Use synthetic division to show that $x = 1 + i$ is a solution of the equation $f(x) = 0$.
   c. (5 pts) Find the linear factorization of f that is promised to us in the Fundamental Theorem of Algebra.

9. Let $z = 8 - 8i$
   a. (5 pts) Find $z + \bar{z}$ and $z\bar{z}$, where $\bar{z}$ is the complex conjugate of z.
   b. (5 pts) Express z in trigonometric form.

10. Let $z = 16\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$.
   a. (5 pts) Express z in standard form.
   b. (5 pts) Find the principal 4th root of z, i.e., find $\sqrt[4]{z}$. Leave z in trigonometric form for this.
   c. (5 pts) Now, find *all* the 4th roots of z, in trigonometric form.
   d. (5 pts) Find the trigonometric form of z^2.
   e. (5 pts) Finally, let $w = 3\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$, and find the trigonometric form of the product $z \cdot w$.

Answer as many as you have time for! Woo-Hoo!

B1 (5 pts) Find the area of the triangle in the 1st problem.

B2 A gun with a muzzle velocity of 370 meters per second is fired, with an angle of 15^0 from the horizontal.
   a. (5 pts) Find the horizontal and vertical components of the bullet, as it leaves the muzzle, accurate to 4 decimal places.
   b. (5 pts) Use a half-angle formula to find the *exact* value for the answer to the previous.
   c. (5 pts) Using - 9.8 $\frac{m}{s^2}$ for the acceleration due to gravity, and neglecting air friction, predict where and when the bullet will hit the ground, in the gun question.

B3 (5 pts) Find $\sin(2u), \cos(2u)$ and $\tan(2u)$, given that $\cos(u) = -\frac{2}{5}$ and $\sin(u) < 0$. Use the 1st two answers to *build* the 3rd. It's *silly* to go back to your cheat sheet and deal with the mess.

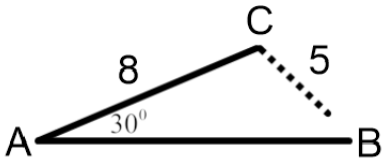
B4 (5 pts) Find $\sin\left(\frac{u}{2}\right), \cos\left(\frac{u}{2}\right)$ and $\tan\left(\frac{u}{2}\right)$, given that $\cos(u) = -\frac{2}{5}$ and $\sin(u) < 0$. Use the 1st two answers to *build* the 3rd. It's *silly* to go back to your cheat sheet and deal with the mess.

B5 (5 pts) Build a sine function that achieves its maximum height of $y = 62$ meters at time $x = 5$ seconds and its minimum height of $y = -8$ meters at $x = 13$ seconds.

B6 (5 pts) Find all solutions of the equation $2\sin^2(3x) - 1 = 0$ in the interval $[0, 2\pi)$.

B7 (5 pts) Sketch the graph of $4\sin\left(\frac{2\pi}{7}x - \frac{26\pi}{7}\right) - 11$.

B8 The triangle described has 2 possible solutions: (See Figure on Right)
Angle $A = 30^0$, side $b = 8$ and side $a = 5$.



a. (5 pts) Prove there are 2 possible triangles from this ambiguous information.
b. (5 pts) Find both triangles.
c. (5 pts) Use your work to find the area of both triangles.