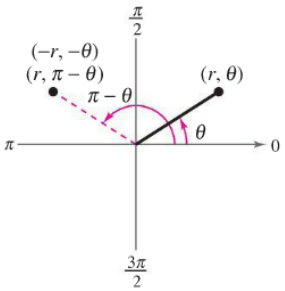
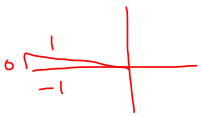
Section 6.8 Graphs of Polar Equations

As usual, graphing by just plotting points is the last-ditch effort of a person who has no clue what the thing looks like. We want as much insight/intuition as possible to help guide us, starting with the different kinds of symmetry.

Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.

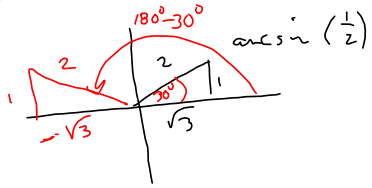


Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$



$\sin x = \frac{1}{2}$

$x = \arcsin\left(\frac{1}{2}\right)$ is half of it.



$r = f(\theta) = 1 + \sin\theta$

$-r = 1 + \sin(-\theta)$

$-r = 1 - \sin\theta$

$r = \sin\theta - 1$

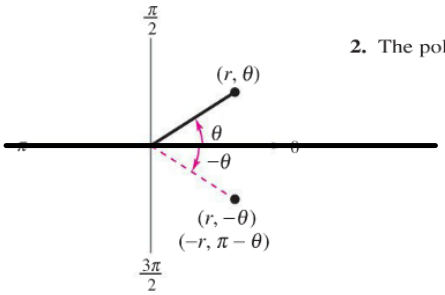
$r = 1 + \sin(\pi - \theta)$

$= 1 + (\sin\pi)(\cos(-\theta))$

$+ \sin(-\theta)(\cos\pi)$

$= 1 + 0 + (-\sin\theta)(-1)$

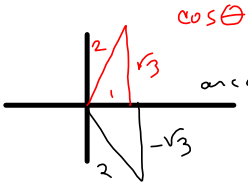
$= 1 + \sin\theta$!



Symmetry with Respect to the Polar Axis

2. The polar axis: Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.

$\cos\theta = \frac{1}{2}$



$\arccos\left(\frac{1}{2}\right)$

$r = 1 + \cos\theta$

$r = 1 + \cos(-\theta)$

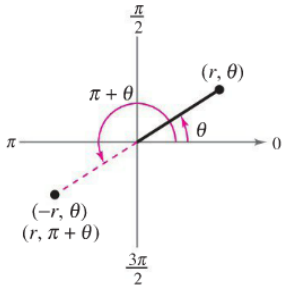
$r = 1 + \cos\theta$ Yes!

Sym. wrt polar axis.

$-r = 1 + \cos(-\theta)$

$-r = 1 + \cos\theta$

Nope.



Symmetry with Respect to the Pole

3. The pole:

Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.

rose

$r = 5\sin(2\theta)$

$r = 5\sin(2(\pi + \theta))$

$= 5\sin(2\pi + 2\theta)$

$= 5\sin(2\theta)$

Yup.

lemniscate.

$r^2 = 4\sin^2\theta$

$r^2 = 4\sin^2(\pi + \theta)$

$= 4\left(\frac{1 - \cos(2(\theta + \pi))}{2}\right)$

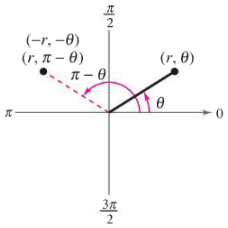
$= 4\left(\frac{1 - \cos\theta}{2}\right)$

$= 4\sin^2\theta$

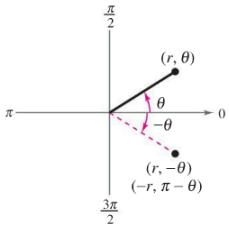
Yup

$(-r)^2 = r^2 = 4\sin^2\theta$

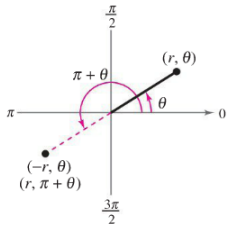
Summary



Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$



Symmetry with Respect to the Polar Axis



Symmetry with Respect to the Pole

Tests for Symmetry in Polar Coordinates

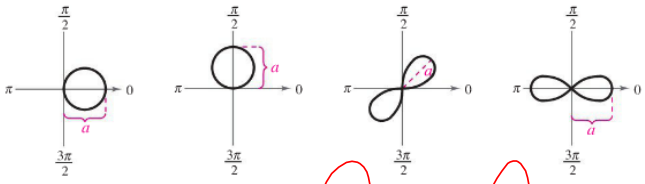
The graph of a polar equation is symmetric with respect to the following when the given substitution yields an equivalent equation.

1. The line $\theta = \pi/2$: Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.
2. The polar axis: Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.
3. The pole: Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.

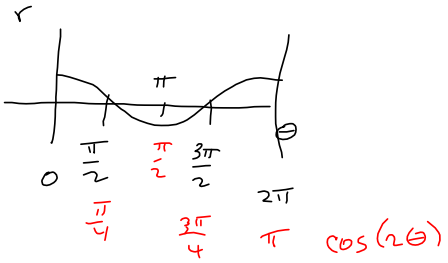
Quick Tests for Symmetry in Polar Coordinates

1. The graph of $r = f(\sin \theta)$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.
2. The graph of $r = g(\cos \theta)$ is symmetric with respect to the polar axis.

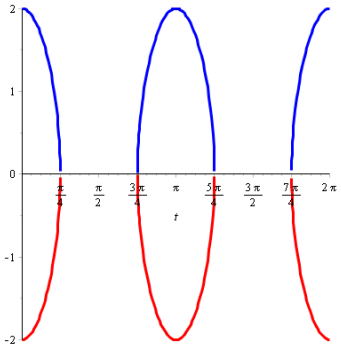
Circles and Lemniscates



$r = a\cos\theta$	$r = a\sin\theta$	$r^2 = a^2\sin 2\theta$	$r^2 = a^2\cos 2\theta$
Circle	Circle	Lemniscate	Lemniscate

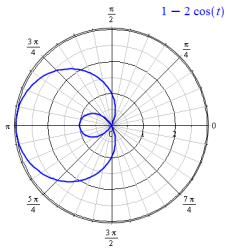


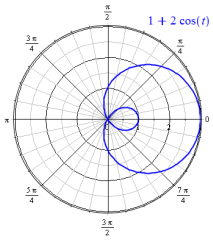
$r^2 = 4\cos(2\theta)$



$r = 2\sqrt{\cos(2\theta)}$, $r = -2\sqrt{\cos(2\theta)}$

limaçon w/ loop





$1 - 2\cos t = 0$

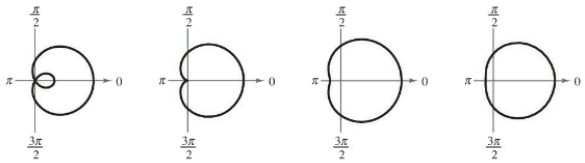
$2\cos t = 1$

$\cos t = \frac{1}{2}$

cedilla

Limaçons

$r = a \pm b\cos\theta,\ r = a \pm b\sin\theta \quad (a > 0, b > 0)$



$\frac{a}{b} < 1$	$\frac{a}{b} = 1$	$1 < \frac{a}{b} < 2$	$\frac{a}{b} \geq 2$
Limaçon with inner loop	Cardioid (heart-shaped)	Dimpled limaçon	Convex limaçon

$1 + \cos\theta$

$1 + \sin\theta$

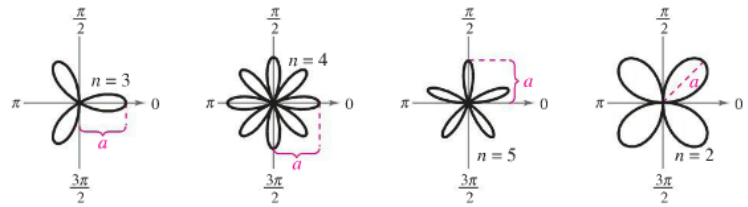
$3 + 2\cos\theta$

$4 + 2\sin\theta$

$5 + \sin\theta$

Rose Curves

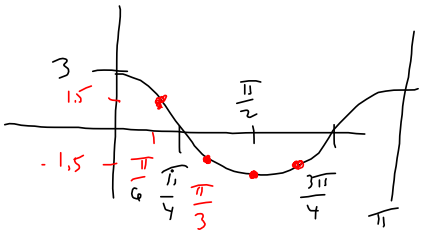
n petals when n is odd, $2n$ petals when n is even $(n \geq 2)$



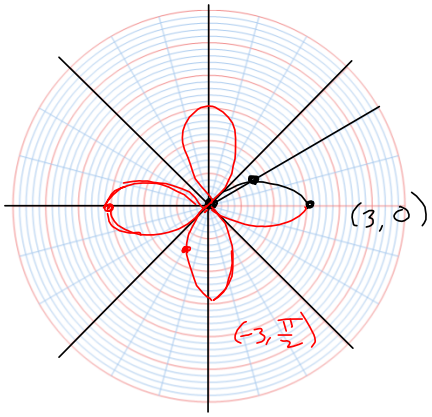
$r = a \cos n\theta$ $r = a \cos n\theta$ $r = a \sin n\theta$ $r = a \sin n\theta$

$2\cos(3\theta)$ $5\cos(4\theta)$

$r = 3\cos(2\theta)$ Symmetry polar axis



π $\frac{5\pi}{4}$ $\frac{3\pi}{2}$ $\frac{7\pi}{4}$ 2π



LarTrig9 6.8.001. (2456909) (Add) -- view
Comment: not randomized

Fill in the blank.

The graph of $r = f(\sin\theta)$ is symmetric with respect to the line _____.

- $\theta = \frac{\pi}{2}$

$r = \sin^2\theta + 17\sin\theta$

LarTrig9 6.8.002. (2456656) (Add) -- view
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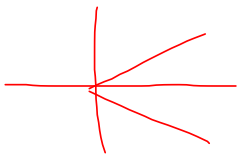
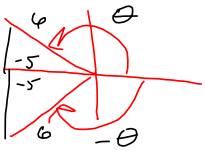
Fill in the blank.

The graph of $r = g(\cos(\theta))$ is symmetric with respect to the ---Select--- polar axis

$r = \cos^2\theta - 23\cos\theta$





Nov 28-11:52 AM

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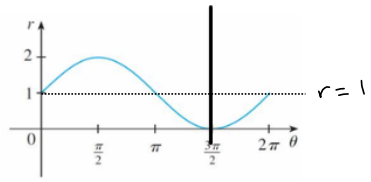
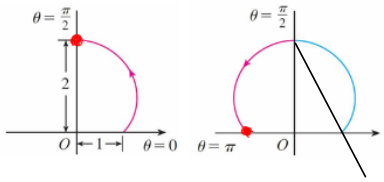
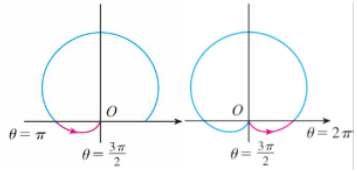
1m

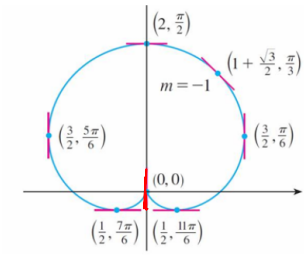
Fill in the blank.

The equation $r = 1 + \sin\theta$ represents a ---Select--- cardioid .









General
limaçon
specifically
cardioid a=b
$a+b\sin\theta$

Nov 29-9:44 AM

LarTrig9 6.8.004. (2456211) (Remove) -- view

1m

Fill in the blank.

The equation $r = 3 \cos \theta$ represents a ---Select---

LarTrig9 6.8.005. (2456911) (Remove) -- view

Comment: not randomized

1m

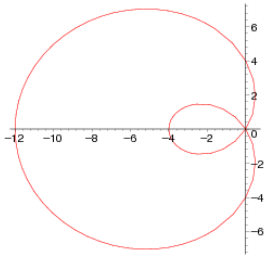
Fill in the blank.

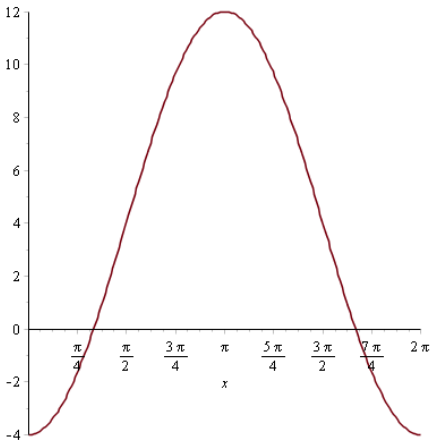
The equation $r^2 = 4 \sin 2\theta$ represents a ---Select---

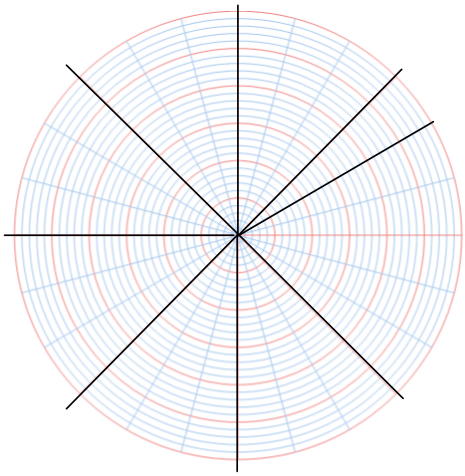
LarTrig9 6.8.009. (2456922) (Remove) -- view

Identify the type of polar graph.

$r = 4(1 - 2\cos(\theta))$



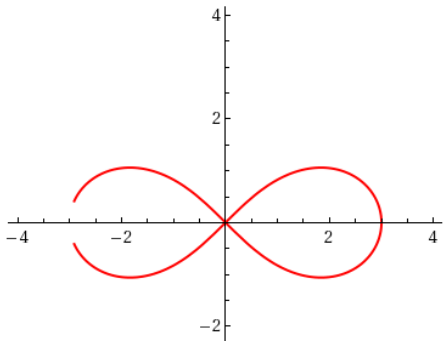


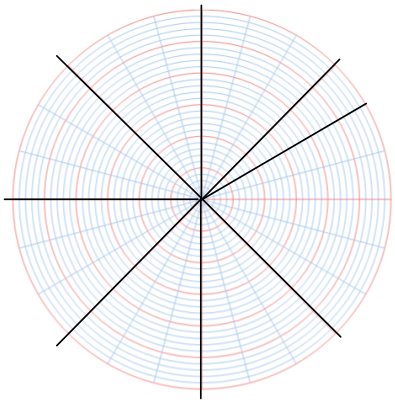


LarTrig9 6.8.010. (2534285) (Add) -- view

Identify the type of polar graph.

$r^2 = 9 \cos 2\theta$

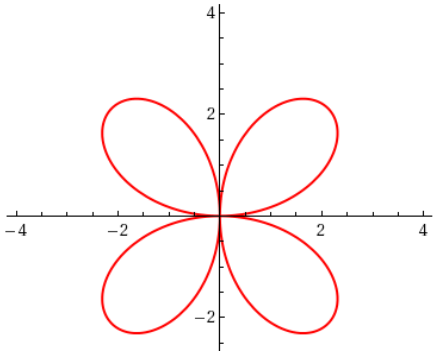


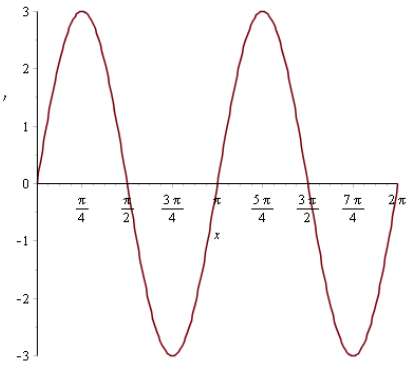


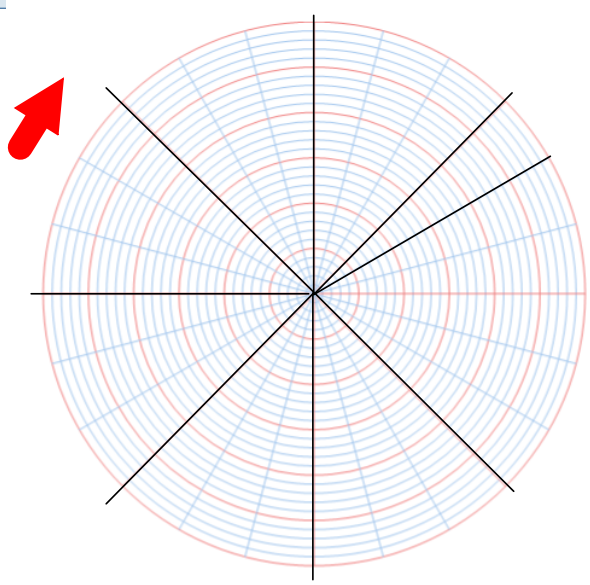
Nov 29-9:54 AM

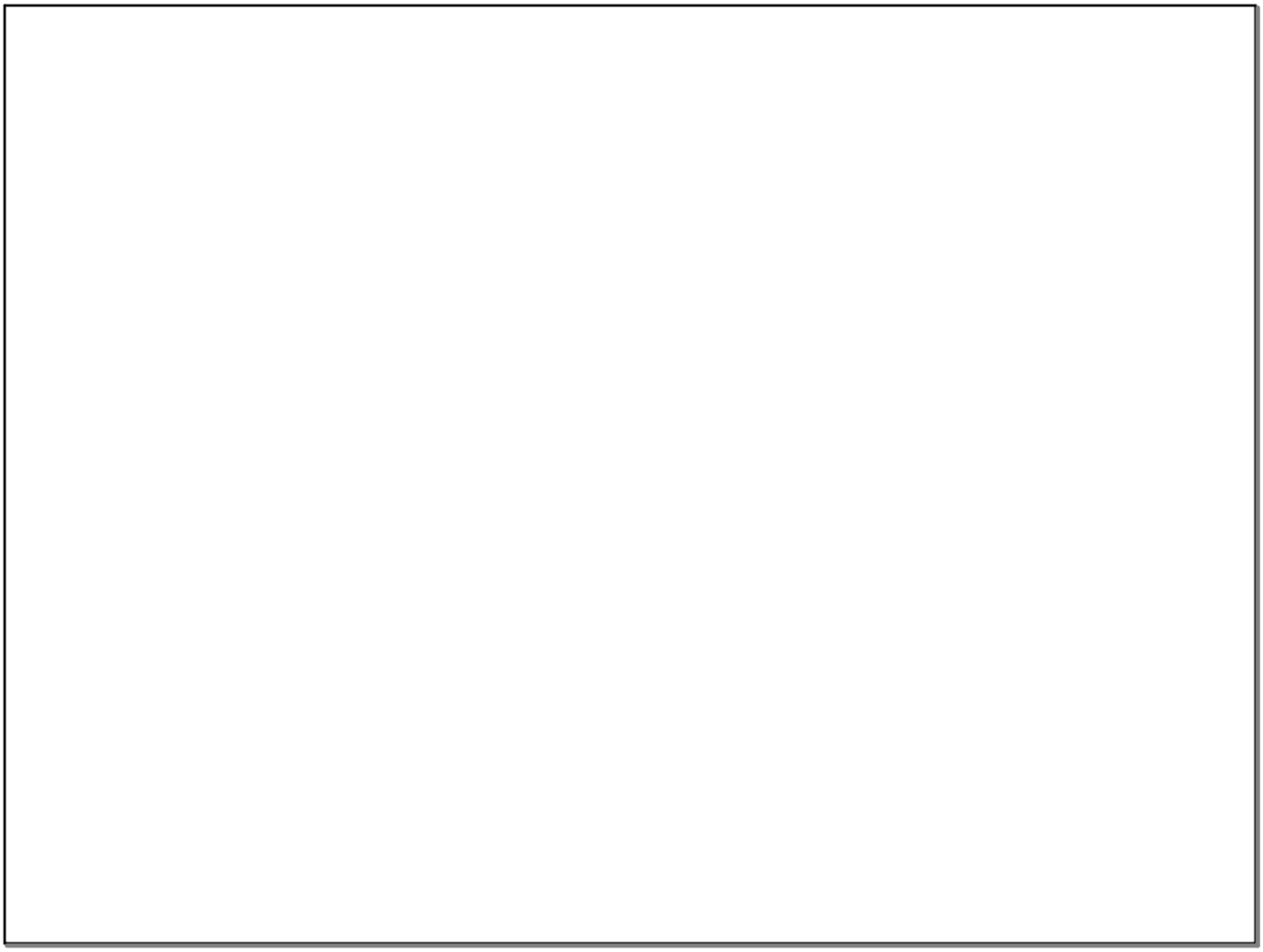
Identify the type of polar graph.

$r = 3 \sin 2\theta$









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