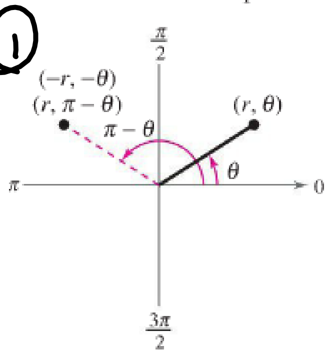
Tests for Symmetry in Polar Coordinates

The graph of a polar equation is symmetric with respect to the following when the given substitution yields an equivalent equation.

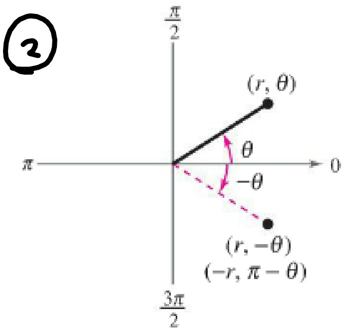
1. The line $\theta = \pi/2$: Replace (r, θ) by $(r, \pi - \theta)$ or $(-r, -\theta)$.
2. The polar axis: Replace (r, θ) by $(r, -\theta)$ or $(-r, \pi - \theta)$.
3. The pole: Replace (r, θ) by $(r, \pi + \theta)$ or $(-r, \theta)$.

1



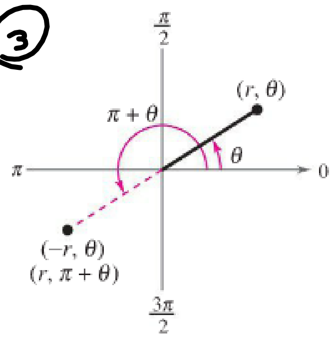
Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$

2



Symmetry with Respect to the Polar Axis

3



Symmetry with Respect to the Pole

$r = 3 + 2\cos\theta$

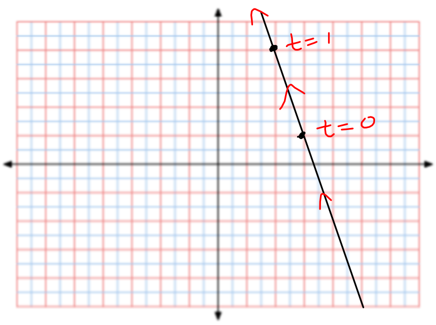
6. Question Details LarTrig9 6.6.010.M

Consider the following.

$x = 6 - 2t$
$y = 2 + 6t$

(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

t	x	y
0	6	2
1	4	8
2	2	14



No restriction on domain if t is unrestricted.

(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

Adjust the domain of the rectangular equation, if necessary.

$$x = 6-2t \Rightarrow x-6=-2t \Rightarrow \frac{x-6}{-2}=t \Rightarrow$$

$$y = 2+6t = 2+6\left(\frac{x-6}{-2}\right) = 2-3(x-6) = 2-3x+18$$

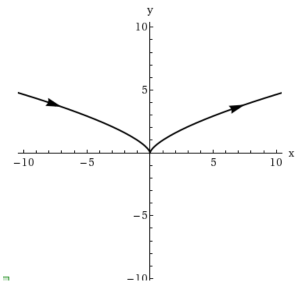
$$\boxed{y = -3x+20}$$

7. LarTrig9 6.6.017. (2519563) *ndomized*

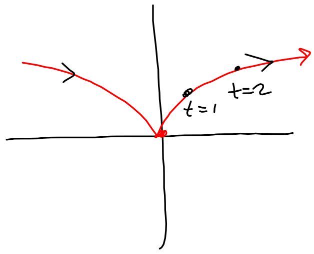
Consider the following.

$x = t^3$
$y = t^2$

(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

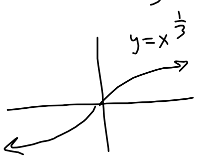


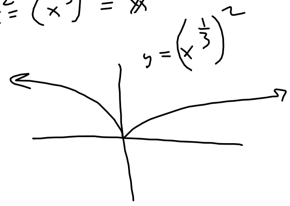
t	x	y
0	0	0
1	1	1
2	8	
3	27	9
−1	−1	1
−2	−8	4



$x = t^3 \Rightarrow \sqrt[3]{x} = t$

$\Rightarrow y = t^2 = (x^{\frac{1}{3}})^2 = x^{2/3}$





(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

7b $y = x^{2/3}$

Adjust the domain of the rectangular equation, if necessary. not necessary

8. LarTrig9 6.6.020. (2534247)

Consider the following.

$x = t - 1$

$y = \dfrac{t}{t-1}$

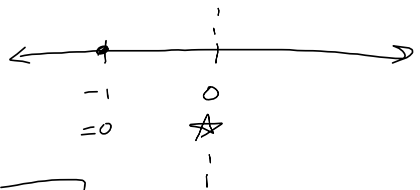
(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

t	x	y
0	−1	0
1	0	*
2	1	2
3	2	$\frac{3}{2}$
−1		
−2		
−3		

Ugh. Sucks graphing parametrically. Much more fun to eliminate the parameter, for this one.

$x = t - 1 \Rightarrow t = x + 1 \Rightarrow$

$y = \dfrac{t}{t-1}$ $\quad y = \dfrac{x+1}{x}$



H.A.: Horizontal Asymptote:

as $x \to \pm\infty$:

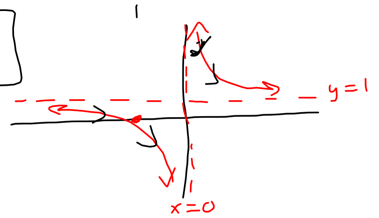
$\dfrac{x+1}{x} = \dfrac{\left(1 + \frac{1}{x}\right)x}{x} = 1 + \frac{1}{x}$

$\xrightarrow{x \to \pm\infty}$ $\boxed{1 = y}$

So, "+" to for right / left.

V.A.: Vertical asymptote:

$x = 0$



(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

8b $y = \dfrac{x+1}{x}$

Adjust the domain of the rectangular equation, if necessary. all real numbers $x \neq 0$

Unsatisfactory. Either $\mathbb{R} \setminus \{0\}$ or

$\mathcal{D} = (-\infty, 0) \cup (0, \infty)$

Domain is a SET, not a condition.

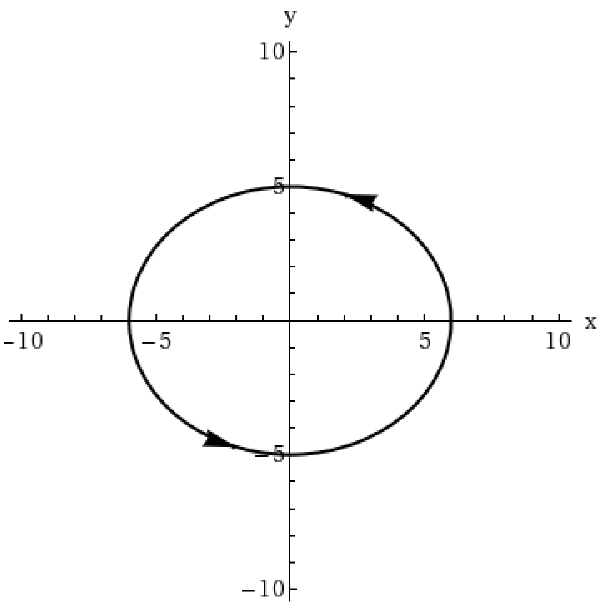
9. LarTrig9 6.6.023. (2519573)

Consider the following.

Ugh! I'd rather eliminate the parameter, with just enough "parametric" to get a direction.

$x = 6 \cos \theta$
$y = 5 \sin \theta$

(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

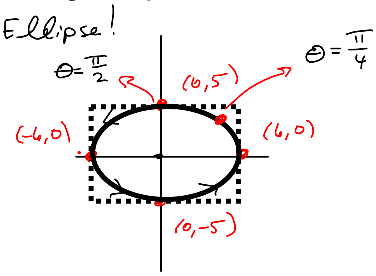


$\frac{x}{6} = \cos\theta$

$\frac{y}{5} = \sin\theta$

So $\left(\frac{x}{6}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$

Ellipse!



(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

9b $\boxed{\dfrac{x^2}{36} + \dfrac{y^2}{25} = 1}$

Adjust the domain of the rectangular equation, if necessary. [−6, 6]

10. LarTrig9 6.6.027. (2534204)

Consider the following.

$x = 1 + \cos\theta$
$y = 1 + 2\sin\theta$

Solve for $\sin\theta$ & $\cos\theta$, then do Pythagorus.

$x - 1 = \cos\theta$

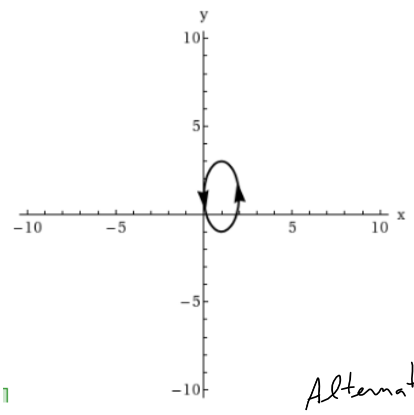
$\frac{y-1}{2} = \sin\theta$

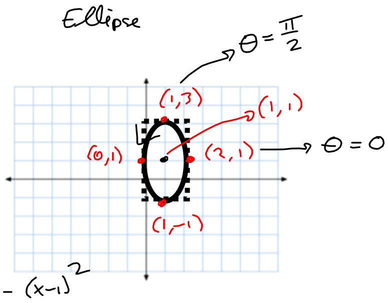
(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

$(x-1)^2 + \left(\frac{y-1}{2}\right)^2 = 1$

$(x-1)^2 + \frac{(y-1)^2}{4} = 1$

Ellipse





Alternate: $\frac{(y-1)^2}{4} = 1 - (x-1)^2$

$(y-1)^2 = 4(1-(x-1)^2)$

$y - 1 = \pm 2\sqrt{1-(x-1)^2}$

$y = 1 \pm 2\sqrt{1-(x-1)^2}$

Domain: $1-(x-1)^2 \geq 0$

$1 - x^2 + 2x - 1 = -x^2 + 2x \geq 0$

N y y y N
− + −
0 2

$= [0,2] = \mathcal{D}$.

(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

10b $\frac{(x-1)^2}{1} + \frac{(y-1)^2}{4} = 1$

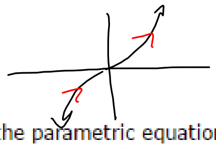
Adjust the domain of the rectangular equation, if necessary. [0, 2] See pic! and/or see "Domain" above.

11. LarTrig9 6.6.030. (2524409)

Consider the following.

$x = e^t$

$y = e^{3t} = (e^t)^3 = x^3$



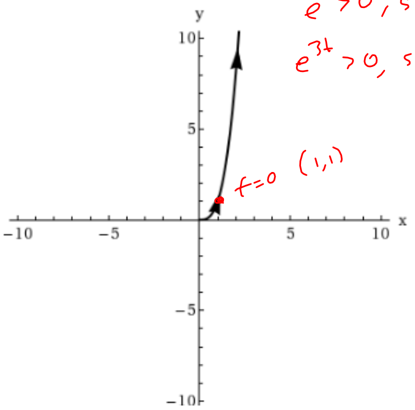
(a) Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

$e^t > 0$, so $x > 0$!

$e^{3t} > 0$, so $y > 0$!

$x = e^t$ $\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = (0, \infty)$



(b) Eliminate the parameter and write the resulting rectangular equation whose graph represents the curve.

11b $y = x^3$

$y = x^3$

Adjust the domain of the rectangular equation, if necessary. $(0, \infty)$

$\{x \mid x > 0\}$

$= (0, \infty)$

12. LarTrig9 6.6.050.MI. (2456330)

Eliminate the parameter and obtain the standard form of the rectangular equation.

Circle: $x = h + r\cos\alpha,\ \ y = k + r\sin\alpha$

12a $(x-h)^2+(y-k)^2=r^2$

(h,k) = center

r = radius

Pythagorus

$\frac{x-h}{r} = \cos\alpha$

$\frac{y-k}{r} = \sin\alpha$

$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$

$(x-h)^2+(y-k)^2=r^2$

3. LarTrig9 6.6.051. (2534688) *randomized*

Eliminate the parameter and obtain the standard form of the rectangular equation.

Ellipse: $x = h + a\cos\theta,\ y = k + b\sin\theta$

13a $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

$\frac{x-h}{a} = \cos\theta \qquad \frac{y-k}{b} = \sin\theta$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

14. LarTrig9 6.6.052. (2456135)

Eliminate the parameter and obtain the standard form of the rectangular equation.

Hyperbola: $x = h + a\sec(\alpha),\ \ y = k + b\tan(\alpha)$

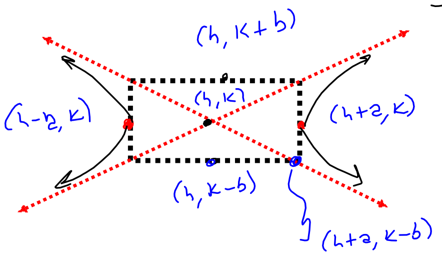
14a $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Recall: $\sec^2\alpha = \tan^2\alpha + 1$, so

$\sec^2\alpha - \tan^2\alpha = 1$

$\frac{x-h}{a} = \sec\alpha \ , \quad \frac{y-k}{b} = \tan\alpha$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$



$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

you can build eqns of the asymptotes, if you want.

BUILD THE BOX. Know which way it opens

Apr 25-11:37 AM