

LarTrig9 4.4.001. (2456705) (Remove) -- view

(1m)

Fill in the blank.

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DeMoivre's Theorem states that if $z = r(\cos \theta + i \sin \theta)$ is a complex number and n is a positive integer, then $z^n = r^n(\cos n\theta + i \sin n\theta)$.

LarTrig9 4.4.002. (2524831) (Add) -- view

(1m)

Fill in the blank.

The complex number $u = a + bi$ is an nth root - or - nth root of the complex number z when $z = u^n = (a + bi)^n$.

$$\begin{aligned} & r_1 (\cos \theta_1 + i \sin \theta_1) \cdot r_2 (\cos \theta_2 + i \sin \theta_2) \\ &= r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)) = r^n (\cos(n\theta_1) + i \sin(n\theta_1)) \end{aligned}$$

modulus $\sqrt[n]{r}$
Angle θ/n

$$\begin{aligned} z_1 \star z_2 &= r_1 (\cos \theta_1 + i \sin \theta_1) \star r_2 (\cos \theta_2 + i \sin \theta_2) \\ &= r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)) \end{aligned}$$

LarTrig9 4.4.003. (2456824) (Add) -- view

(2m)

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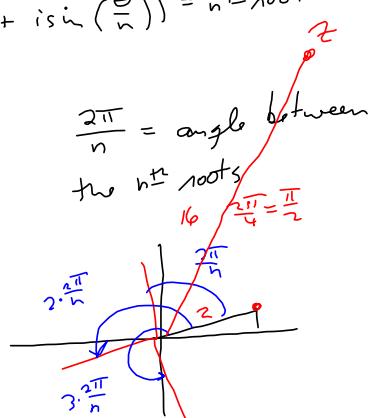
Fill in the blank.

For a positive integer n , the complex number $z = r(\cos \theta + i \sin \theta)$ has exactly n distinct n th roots given by _____, where $k = 0, 1, 2, \dots, n-1$.

$\sqrt[n]{r} \left(\cos \left(\frac{\theta + 2\pi k}{n} \right) + i \sin \left(\frac{\theta + 2\pi k}{n} \right) \right)$

$$r^{\frac{1}{n}} \left(\cos \left(\frac{\theta}{n} + \frac{2\pi k}{n} \right) + i \sin \left(\frac{\theta}{n} + \frac{2\pi k}{n} \right) \right)$$

principal
 $\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \left(\frac{\theta}{n} \right) + i \sin \left(\frac{\theta}{n} \right) \right) = \text{nth root}$
Other nth roots



LarTrig9 4.4.004. (2456513) (Add) -- view

(2m)

Comment: not randomized

Fill in the blank.
The n distinct n th roots of 1 are called the n th roots of unity.

If $z = a+bi$ is 5th root of unity,
then if $v = c+di$ is another complex #,

then zv rotates v counter-clockwise by

an angle = the angle for z

z is 5th root of unity, say the principal one.

then angle for z is $\frac{\pi}{5}$, $|z|=1$

and zv rotates v by $\frac{\pi}{5}$

LarTrig9 4.4.006.MI. (2456937) (Add) -- view 9m

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$$\begin{aligned} z^2 &= (2 + 2i)^2 \\ &\quad \text{Diagram: } r = \sqrt{2^2 + 2^2} = \sqrt{8} \\ &\quad \theta = \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} z^2 &= \left(2\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right)^2 \\ &= (2\sqrt{2})^2 \left(\cos(2 \cdot \frac{\pi}{4}) + i \sin(2 \cdot \frac{\pi}{4})\right) \\ &= 8 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 8i \end{aligned}$$

Handwritten note: circled 8i with a red arrow pointing to it.

LarTrig9 4.4.008. (2456202) (Add) -- view 5m

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$$z^8 = (3 - 2i)^8$$

$$\begin{aligned} r &= \sqrt{3^2 + (-2)^2} \\ &= \sqrt{9+4} = \sqrt{13} \end{aligned}$$

$$\begin{aligned} \theta' &= \arctan\left(\frac{-2}{3}\right) = -\arctan\left(\frac{2}{3}\right) \approx -0.5880026035 \approx \theta \\ z^8 &= (\sqrt{13})^8 \left(\cos(-0.5880026035) + i \sin(-0.5880026035)\right) \\ &\approx 28561 (-0.008360543 + 0.999964987i) \\ &\approx -239.000 + 28560.000i \end{aligned}$$

Handwritten note: circled 28561 and -239.000 with a red arrow pointing to them. A bracket groups these with the text "8 times it is".

$\cos(\text{Ans})$ -0.0083680543	$-\tan^{-1}(2/3)*8$ -4.704020828	$\sin(\text{Ans})$ 0.9999649872
$-\tan^{-1}(2/3)*8$ -4.704020828	13^{4*8} 13^4*(-.0083680543+i*Ans)	13^{4*8} 13^4*(-.0083680543+i*Ans)
$\sin(\text{Ans})$ 0.9999649872	$-238.9999989+28560i$	$....9999989+28560i$

LarTrig9 4.4.009. (2534269) (Add) -- view -

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$$3(\sqrt{3} + i)^{10}$$

A large red arrow points from the input field to the output field.

LarTrig9 4.4.012.MI. (2456207) (Add) -- view

6m



Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$$\left[3 \left(\cos 60^\circ + i \sin 60^\circ \right) \right]^4$$

LarTrig9 4.4.013. (2534261) (Remove) -- view

10m



Use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

$$\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)^{12}$$

LarTrig9 4.4.030. (2456199) (Remove) -- view

10m



Find the square roots of the complex number. (Enter your answers as a comma-separated list.)

3i

$$\frac{\sqrt{6} + i\sqrt{6}}{2}, \frac{-\sqrt{6} - i\sqrt{6}}{2}$$

$$\begin{aligned} & \sqrt{3} \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right) \quad \frac{2\pi}{2} = \pi \\ & \sqrt{3} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) \\ & = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}} i = \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2} i \\ & \sqrt{3} \left(\cos \left(\frac{5\pi}{4} \right) + i \sin \left(\frac{5\pi}{4} \right) \right) \quad \frac{\pi}{4} + \pi = \frac{5\pi}{4} \end{aligned}$$

$$\begin{aligned} & -\sqrt{6} - \frac{\sqrt{6}}{2} i \\ & -\frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{2} i \end{aligned}$$

LarTrig9 4.4.035. (2537351) (Add) -- view

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10m



Find the square roots of the complex number. (Enter your answers as a comma-separated list.)

$$1 + \sqrt{3}i$$

LarTrig9 4.4.039. (2534305) (Add) -- view



Consider the following.

$$\text{Cube roots of } 8 \left(\cos \frac{4\pi}{9} + i \sin \frac{4\pi}{9} \right)$$

(a) Use this [formula](#) to find the indicated roots of the complex number. (Enter your answers in trigonometric form.)

$k = 0$

$k = 1$

$k = 2$

(b) Represent each of the roots graphically.

LarTrig9 4.4.042. (2524825) (Add) -- view



Consider the following.

$$\text{Fifth roots of } 32 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) = z$$

$$r^{\frac{1}{5}} = 32^{\frac{1}{5}} = 2$$

$$\frac{5\pi}{6} = \frac{\pi}{6}$$

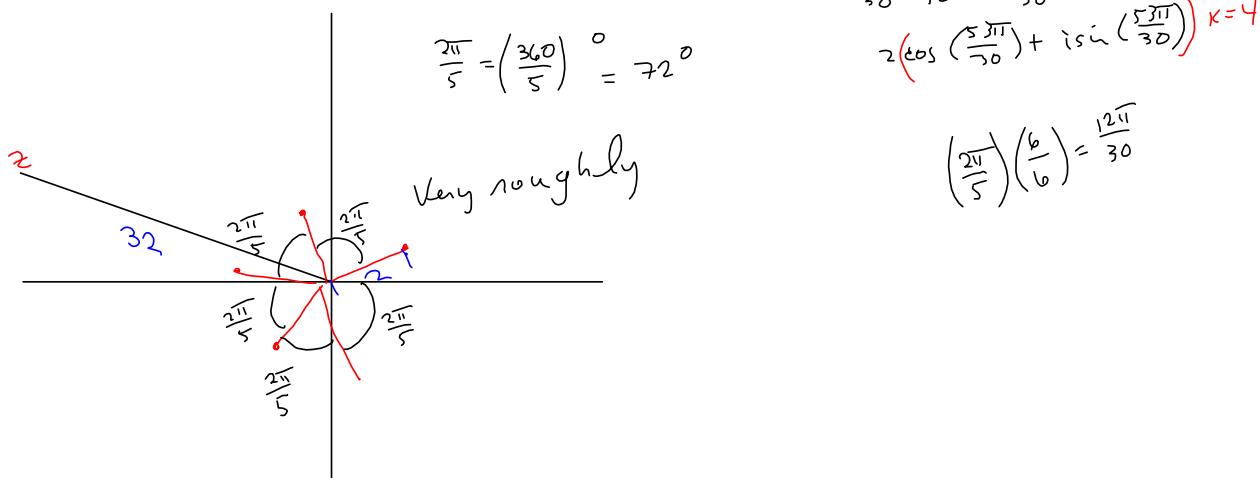
(a) Use this formula to find the indicated roots of the complex number. (Enter your answers in trigonometric form.)

$k = 0$	$\sqrt[5]{z}$	$2 \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right)$	$2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$
$k = 1$		$2 \left(\cos \left(\frac{17\pi}{30} \right) + i \sin \left(\frac{17\pi}{30} \right) \right)$	$= 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt{3} + i \quad k=0$ $\frac{2\pi}{5}$ is our increment.
$k = 2$		$2 \left(\cos \left(\frac{29\pi}{30} \right) + i \sin \left(\frac{29\pi}{30} \right) \right)$	$\frac{17\pi}{30} + \frac{2\pi}{5} = \frac{5\pi + 12\pi}{30} = \frac{17\pi}{30}$
$k = 3$		$2 \left(\cos \left(\frac{41\pi}{30} \right) + i \sin \left(\frac{41\pi}{30} \right) \right)$	$2 \left(\cos \frac{17\pi}{30} + i \sin \frac{17\pi}{30} \right) \quad k=1$
$k = 4$		$2 \left(\cos \left(\frac{53\pi}{30} \right) + i \sin \left(\frac{53\pi}{30} \right) \right)$	$\frac{17\pi}{30} + \frac{2\pi}{5} = \frac{29\pi}{30}$ $2 \left(\cos \left(\frac{29\pi}{30} \right) + i \sin \left(\frac{29\pi}{30} \right) \right) \quad k=2$

(b) Represent each of the roots graphically.

$$\frac{2\pi}{5} = \left(\frac{360}{5} \right)^\circ = 72^\circ$$

Very roughly



$$\left(\frac{2\pi}{5} \right) \left(\frac{6}{5} \right) = \frac{12\pi}{30}$$

$$\frac{41\pi}{30} + \frac{12\pi}{30} = \frac{53\pi}{30}$$

$$2 \left(\cos \left(\frac{53\pi}{30} \right) + i \sin \left(\frac{53\pi}{30} \right) \right) \quad k=4$$

[LarTrig9 4.4.044. \(2524827\) \(Add\)](#) -- view

17m



Consider the following.

Fourth roots of $256i$

(a) Use this [formula](#) to find the indicated roots of the complex number. (Enter your answers in trigonometric form.)

$k = 0$

$k = 1$

$k = 2$

$k = 3$

(b) Represent each of the roots graphically.

Imaginary axis

Inr