

#6 ^{S4.2} How many complex solutions does
 $x^6 + 4x^2 + 12 = 0$ have?

Fundamental Theorem of Algebra: A polynomial of degree n has at least one complex root (zero).

Factor Theorem: If $x = c$ is a root of a polynomial, then $(x - c)$ is a factor of the polynomial.

$$f(x) = \underline{x^3 - 27}, \quad x=3 \text{ is a root.} \quad \rightarrow \text{degree} = n=3$$

$$= (x-3)(x^2 + 3x + 9)$$

$\rightarrow \text{degree} = n-1 = 2$

$$-1.722448199, 0.8612241000 - 2.495019209i, 0.8612241000 + 2.495019209i$$

$$\curvearrowright u = \curvearrowright$$

$$u = x^2$$

$$x^2 = -1.722448199$$

$$\Rightarrow x = \pm \sqrt{-1.722448199} = \pm i \sqrt{1.722448199}$$

$$u = .8612241000 - 2.495019209i = x^2$$

$$\theta = \arctan\left(-\frac{2.49\dots}{.861\dots}\right)$$



$$\sqrt{(.8612241000)^2 + (-2.495019209)^2} = r$$

Use De Moivre to find
 the square roots of
 the nonreal solutions.