LarTrig9 4.2.001. (2524415) (Remove) -- view

Fill in the blanks.

The ---Select--- Fundamental Theorem of ---Select--- Algebra states that if $f(x)$ is a polynomial of degree n, where $n > 0$, then f has at least one zero in the complex number system.

FACTOR THM: $x = c$ is root $\Rightarrow x - c$ is a factor.

FTA + Factor Theorem $\Rightarrow$ Linear Factorization Thm.

LarTrig9 4.2.002. (2524440) (Remove) -- view

Fill in the blank.

The ---Select--- Linear Factorization Theorem states that if $f(x)$ is a polynomial of degree n, where $n > 0$, then $f(x)$ has precisely n linear factors, $f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$, where $c_1, c_2, \ldots, c_n$ are complex numbers.

FTA: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$

FTA + Factor Theorem

$$= (x - c_1)(a_n x^{n-1} + \ldots)$$
$$= (x - c_1)(x - c_2)(a_n x^{n-2} + \ldots)$$
$$= \ldots = a_n (x - c_1)(x - c_2) \cdots (x - c_n)$$

LarTrig9 4.2.003. (2456357) (Add) -- view

Comment: not randomized

1m

Fill in the blank.

Two complex solutions of the form $a \pm bi$ of a polynomial equation with real coefficients are called ---Select--- conjugates .

LarTrig9 4.2.004. (2456163) (Add) -- view

Comment: not randomized

1m

Fill in the blank.

The expression inside the radical of the Quadratic Formula, $b^2 - 4ac$, is called the ---Select--- discriminant and is used to determine types of solutions of a quadratic equation.

$b^2 - 4ac$

LarTrig9 4.2.006. (2456864) (Add) -- view

Determine the number of solutions of the equation in the complex number system.

$x^8 + 3x^2 + 12 = 0$

8

LarTrig9 4.2.010. (2456747) (Add) -- view 1m

Use the discriminant to determine the number of real solutions of the quadratic equation.

$3x^2 - x - 4 = 0$

2

$a=3,\ b=-1,\ c=-4$

$b^2-4ac = (-1)^2-4(3)(-4) =$ pos. $= 49$

LarTrig9 4.2.012. (2456052) (Add) -- view 3m

Use the discriminant to determine the number of real solutions of the quadratic equation.

$\frac{1}{3}x^2 - 3x + 24 = 0$

0

$a=\frac{1}{3},\ b=-3,\ c=24$

$b^2-4ac = (-3)^2 - 4(\frac{1}{3})(\overset{8}{24})$

$= 9 - 32 = -23 < 0$

So 2 nonreal solutions.

LarTrig9 4.2.018. (2524429) (Add) -- view

Comment: slightly modified

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$5x^2 - 1 = 0$

$x =$ $\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}$

$5x^2 = 1$

$x^2 = \frac{1}{5}$

$x = \pm\sqrt{\frac{1}{5}} = \pm\frac{\sqrt{1}}{\sqrt{5}} = \pm\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

LarTrig9 4.2.019. (2532498) (Add) -- view

Comment: slightly modified

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$(x+4)^2 - 3 = 0$

$x =$ $\sqrt{3} - 4, -4 - \sqrt{3}$

$(x+4)^2 = 3$

$x + 4 = \pm\sqrt{3}$

$x = -4 \pm \sqrt{3}$

$\cancel{x^2 + 4x - 3 = 0}$

$\cancel{x^2 = 3 - 4x}$

$\cancel{x = \pm\sqrt{3-4x}}$

LarTrig9 4.2.022. (2524364) (Add) -- view

Comment: slightly modified

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$9x^2 + 6x + 1 = 0$

$x =$ $-\frac{1}{3}$

$(3x)^2 + 2\cdot(3x)\cdot 1 + 1^2 = (3x+1)^2 = 0 \Rightarrow x = -\frac{1}{3}$

1 real zero (repeated)

$(3x+1)^2 = 0$

$3x + 1 = \pm 0 = 0$

LarTrig9 4.2.024. (2524452) (Add) -- view

Comment: slightly modified

Solve the quadratic equation. (Enter your answers as a comma-separated list.)

$81 + 12x - x^2 = 0 \implies x^2 - 12x - 81 = 0$

$x =$ $6 + 3\sqrt{13}, 6 - 3\sqrt{13}$

$a = 1, b = -12, -81 = c$

$b^2 - 4ac = (-12)^2 - 4(1)(-81)$

$= 144 + 324 = 468$

$\sqrt{468} = 2\cdot 3\sqrt{13}$

$= 6\sqrt{13}$

$2 \,|\, 468$

$2 \,|\, 234$

$3 \,|\, 117$

$3 \,|\, 39$

13

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm 6\sqrt{13}}{2(1)}$

$= \boxed{6 \pm 3\sqrt{13}}$

$x^2 - 12x - 81 =$

$= x^2 - 12x + 6^2 - 36 - 81$

$= (x-6)^2 - 117 = 0$

$(x-6)^2 = 117$

$x - 6 = \pm\sqrt{117} = \pm 3\sqrt{13}$

$\boxed{x = 6 \pm 3\sqrt{13}}$

LarTrig9 4.2.027. (2532496) (Add) -- view

Comment: slightly modified

Solve the polynomial equation. (Enter your answers as a comma-separated list.)

$x^4 - 2x^2 - 3 = 0$

$x =$ $\boxed{-\sqrt{3}, \sqrt{3}, i, -i}$

Quadratic in Form

$u^2 - 2u - 3 = (u-3)(u+1)$

$= (x^2-3)(x^2+1) = 0 \Rightarrow$

$x^2 - 3 = 0$ or $x^2+1=0$

$x^2 = 3$ $\quad$ $x^2 = -1$

$x = \pm\sqrt{3}$ $\quad$ $x = \pm i$

LarTrig9 4.2.032. (2456489) (Add) -- view

Consider the following.

$f(x) = x^3 - 2x^2 - 9x + 18$

(a) Use a graphing utility to graph the function.

Factoring by grouping

$x^2(x-2) - 9(x-2)$

$= (x-2)(x^2-9) = (x-2)(x-3)(x+3) = 0$

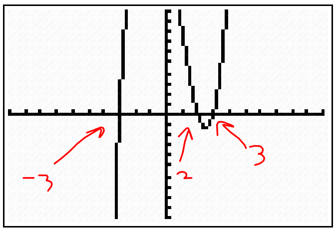
$\Rightarrow x \in \{-3, 3, 2\}$

(b) Find all the zeros of the function.

$x =$ $\boxed{3, -3, 2}$

(c) Describe the relationship between the number of real zeros and the number of x-intercepts of the graph.

3 of each



$\begin{array}{r} -42 \\ 4 \\ \hline 170 \end{array}$

LarTrig9 4.2.036. (2524419) (Add) -- view

Write the polynomial as a product of linear factors.

$f(x) = x^2 - x + 42 \;=\; (x-7)(x+6)$

$f(x) =$ $\left(x + \frac{1}{2}\left(-1 - i\sqrt{167}\right)\right)\left(x + \frac{1}{2}\left(-1 + i\sqrt{167}\right)\right)$

Find all the zeros of the function. (Enter your answers as a comma-separated list.)

$x =$ $\frac{1}{2}\left(1 - i\sqrt{167}\right), \frac{1}{2}\left(1 + i\sqrt{167}\right)$

$x^2 - x = -42$

$x^2 - x + \;= -42 + \frac{1}{4}$

$-\frac{1}{2} \to \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ $\quad \frac{-168+1}{4} = -\frac{167}{4}$

$\left(x - \frac{1}{2}\right)^2 = -\frac{177}{4}$

$f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{167}{4}$

$a = 1, b = -1, c = 42$

$b^2 - 4ac = (-1)^2 - 4(1)(42)$

$= 1 - 168 = -167$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{-167}}{2} = \frac{1 \pm \sqrt{167}\,i}{2}$

$x - \frac{1}{2} = \pm\sqrt{-\frac{167}{4}} = \pm\frac{\sqrt{167}}{2}i$

$x = \frac{1 \pm \sqrt{167}\,i}{2}$

$f(x) = \left(x - \left(\frac{1+\sqrt{167}\,i}{2}\right)\right)\left(x - \left(\frac{1-\sqrt{167}\,i}{2}\right)\right)$

LarTrig9 4.2.047. (2532482) (Add) -- view

$\sqrt{27} = \sqrt{3 \cdot 3 \cdot 3}$

$= 3\sqrt{3}$

Write the polynomial as a product of linear factors.

$f(x) = 2x^3 - x^2 + 54x - 27 \;=\; x^2(2x-1) + 27(2x-1) = (2x-1)(x^2+27)$

$= (2x-1)(x - 3\sqrt{3}\,i)(x + 3\sqrt{3}\,i)$

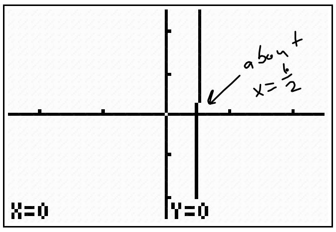
$f(x) =$ $\left(x - 3i\sqrt{3}\right)\left(x + 3i\sqrt{3}\right)(2x-1)$

Find all the zeros of the function. (Enter your answers as a comma-separated list.)

$x =$ $\frac{1}{2}, 3i\sqrt{3}, -3i\sqrt{3}$

Alternate way:

Grapher sez $x = \frac{1}{2}$ is good bet!



Divide by $x - \frac{1}{2}$:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 54 & -27 \\ & & 1 & 0 & 27 \\ \hline & 2 & 0 & 54 & 0 \\ & x^2 & x^1 & c & r \end{array}$$

This says: $\left(x - \frac{1}{2}\right)(2x^2 + 54)$

$= 2\left(x - \frac{1}{2}\right)(x^2 + 27)$

$x^2 + 27 = 0$

$x^2 = -27$

$x = \pm i\,3\sqrt{3}$

X	Y2
.5	0
.5	0

X=.5

Table confirms $x = \frac{1}{2}$ is root

$= 2\left(x - \frac{1}{2}\right)(x - 3\sqrt{3}\,i)(x + 3\sqrt{3}\,i)$

LarTrig9 4.2.057. (2532495) (Add) -- view

Use the given zero to find all the zeros of the function. (Enter your answers as a comma-separated list. Include the given zero in your answer.)

Function	Zero
$g(x) = 3x^3 + 17x^2 + 24x - 10$	$-3 + i$

$x =$ $\frac{1}{3}, -3+i, -3-i$

CPT says $-3-i$ is also a root, b/c all coefficents of $g(x)$ are real

$$\begin{array}{r|rrrr} -3+i & 3 & 17 & 24 & -10 \\ & & -9+3i & -27-i & 10 \\ \hline -3-i & 3 & 8+3i & -3-i & 0 \\ & & -9-3i & 3+i & \\ \hline & 3 & -1 & 0 & \end{array}$$

$(12+3i)(-3+i)$

$= -36 + 12i - 9i + 3i^2$

$= -36 + 3i - 3$

$= -39 + 3i$

Nope!

$(8+3i)(-3+i)$

$= -24 + 8i - 9i + 3i^2$

$= -27 - i$

$(x-(-3+i))(x-(-3-i))(3x-1)$

$3x-1=0$

$3x=1$

$x=\frac{1}{3}$

$(-3+i)(-3-i) = 3^2+i^2 = 10$

LarTrig9 4.2.065. (2537353) (Add) -- view

Find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

2, 3 + *i*

Conjugate pairs theorem

$f(x) =$ $x^3 - 8x^2 + 22x - 20$

$\boxed{(x-2)(x-(3+i))(x-(3-i))}$

$(x-2)(x-3-i)(x-3+i) = (x-2)(x^2-3x+ix-3x+9-3i-ix+3i-i^2)$

$= (x-2)(x^2-6x+9+1)$

$= (x-2)(x^2-6x+10) = x^3-6x^2+10x$

$-2x^2+12x-20$

$x^3-8x^2+22x-20$

LarTrig9 4.2.080. (2456305) (Add) -- view

Find a cubic polynomial function *f* with real coefficients that has the given complex zeros and *x*-intercept. (There are many correct answers.)

Complex Zeros	x-Intercept
$x = -2 \pm \sqrt{2}i$	$(-1, 0)$

$f(x) =$