

1. LarTrig9 3.4.001. (2447292)

Fill in the blank.

The  1a  dot product of two vectors yields a scalar, rather than a vector.  
Dot Product

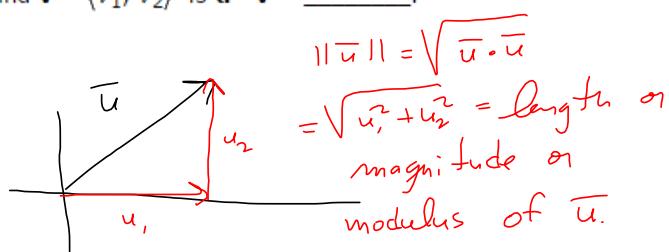
 2. LarTrig9 3.4.002. (2446688)

Fill in the blank.

The dot product of  $\mathbf{u} = \langle u_1, u_2 \rangle$  and  $\mathbf{v} = \langle v_1, v_2 \rangle$  is  $\mathbf{u} \cdot \mathbf{v} = \underline{\hspace{2cm}}$ .

$$u_1 v_1 + u_2 v_2$$

Notice

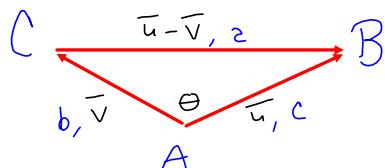


## 3. LarTrig9 3.4.003. (2447395)

Fill in the blank.

If  $\theta$  is the angle between two nonzero vectors  $\mathbf{u}$  and  $\mathbf{v}$ , then  $\cos \theta = \underline{\hspace{2cm}}$ .

- $\mathbf{u} \times \mathbf{v}$
- $\mathbf{u} \cdot \mathbf{v}$  3a
- $\frac{\mathbf{u}}{\|\mathbf{u}\|} + \frac{\mathbf{v}}{\|\mathbf{v}\|}$
- $\mathbf{u} + \mathbf{v}$
- $\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$



Law of Cosines:

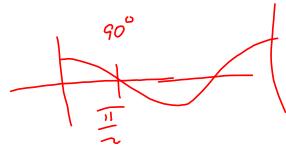
$$z^2 = b^2 + c^2 - 2bc \cos \theta$$

$$\|\mathbf{u} - \mathbf{v}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - 2\|\mathbf{v}\|\|\mathbf{u}\| \cos \theta$$

$$(\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) \\ \cancel{\mathbf{u} \cdot \mathbf{u}} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} + \cancel{\mathbf{v} \cdot \mathbf{v}} = \cancel{\mathbf{v} \cdot \mathbf{v}} + \cancel{\mathbf{u} \cdot \mathbf{u}} - 2\|\mathbf{u}\|\|\mathbf{v}\| \cos \theta$$

$$+ 2\mathbf{u} \cdot \mathbf{v} = + 2\|\mathbf{u}\|\|\mathbf{v}\| \cos \theta \Rightarrow$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

4. LarTrig9 3.4.004. (2595980) 4a orthogonal

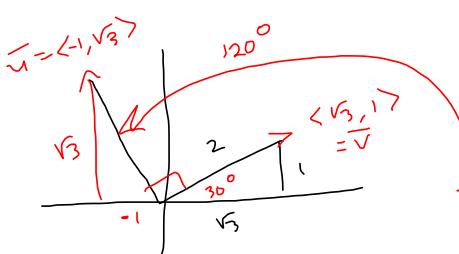
Fill in the blank.

The vectors  $\mathbf{u}$  and  $\mathbf{v}$  are Select 4a orthogonal when  $\mathbf{u} \cdot \mathbf{v} = 0$ 

$$\overrightarrow{i} = \langle 1, 0 \rangle$$

$$\overrightarrow{i} \cdot \overrightarrow{j} = 1 \cdot 0 + 0 \cdot 1 = 0$$

$$\overrightarrow{j} = \langle 0, 1 \rangle$$



$$\overrightarrow{u} \cdot \overrightarrow{v} = \langle 1, 0 \rangle \cdot \langle \sqrt{3}, 1 \rangle$$

$$= (-1)(\sqrt{3}) + (0)(1) = 0$$

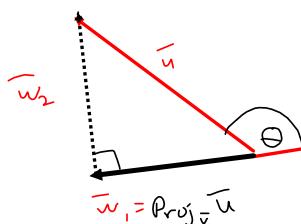
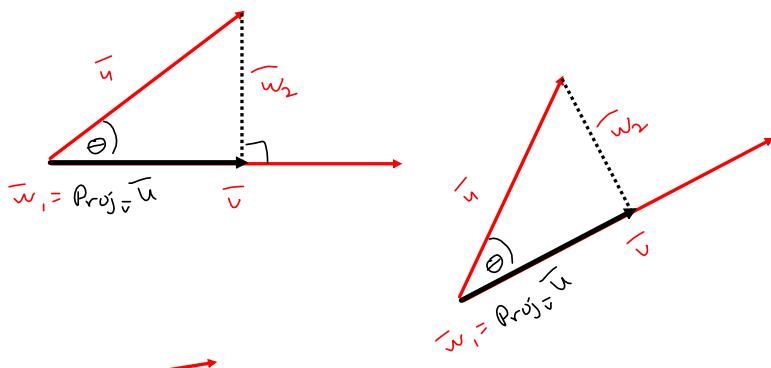
## 5. LarTrig9 3.4.005. (2446204)

Fill in the blank.

The projection of  $\mathbf{u}$  onto  $\mathbf{v}$  is given by  $\text{proj}_{\mathbf{v}} \mathbf{u} = \underline{\hspace{2cm}}$ .

- $\left[ \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right]$
- $\left[ \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2} \right] \mathbf{v}$  5a
- $\left[ \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2} \right] \mathbf{u}$
- $\left[ \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right] \mathbf{u}$
- $\left[ \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \right] \mathbf{v}$

$$\overline{\mathbf{u}} = \overline{\mathbf{w}}_1 + \overline{\mathbf{w}}_2$$



$$\begin{aligned} \overline{\mathbf{u}} &= \overline{\mathbf{w}}_1 + \overline{\mathbf{w}}_2 \\ &= c \overline{\mathbf{v}} + \overline{\mathbf{w}}_2 \end{aligned}$$

$$\overline{\mathbf{v}} \cdot \overline{\mathbf{v}} = \|\overline{\mathbf{v}}\|^2$$

$$\begin{aligned} \overline{\mathbf{u}} \cdot \overline{\mathbf{v}} &= c \overline{\mathbf{v}} \cdot \overline{\mathbf{v}} + \overline{\mathbf{w}}_2 \cdot \overline{\mathbf{v}} \\ &= c \overline{\mathbf{v}} \cdot \overline{\mathbf{v}} \end{aligned}$$

$$\Rightarrow \frac{\overline{\mathbf{u}} \cdot \overline{\mathbf{v}}}{\overline{\mathbf{v}} \cdot \overline{\mathbf{v}}} = c$$

$$\text{So, } \text{proj}_{\overline{\mathbf{v}}} \overline{\mathbf{u}} = \frac{\overline{\mathbf{u}} \cdot \overline{\mathbf{v}}}{\|\overline{\mathbf{v}}\|^2} \overline{\mathbf{v}}$$

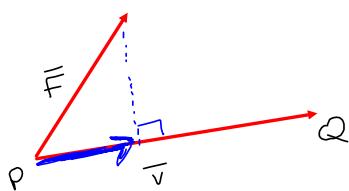
$$= \frac{\overline{\mathbf{u}} \cdot \overline{\mathbf{v}}}{\|\overline{\mathbf{v}}\|} \frac{\overline{\mathbf{v}}}{\|\overline{\mathbf{v}}\|}$$

## 6. LarTrig9 3.4.006. (244614)

Fill in the blank.

The work  $W$  done by a constant force  $\mathbf{F}$  as its point of application moves along the vector  $\vec{PQ}$  is given by \_\_\_\_\_.

- $W = \|\text{proj}_{\vec{PQ}} \mathbf{F}\| \|\mathbf{F}\|$  or  $W = \mathbf{F} \cdot \frac{\vec{PQ}}{\|\vec{PQ}\|}$
- $W = \|\vec{PQ}\|$  or  $W = \mathbf{F} + \vec{PQ}$  [6a]
- $W = \|\text{proj}_{\vec{PQ}} \mathbf{F}\| \|\vec{PQ}\|$  or  $W = \mathbf{F} \times \vec{PQ}$
- $W = \|\text{proj}_{\vec{PQ}} \mathbf{F}\|$  or  $W = \vec{PQ} \times \mathbf{F}$
- $W = \|\text{proj}_{\vec{PQ}} \mathbf{F}\| \|\vec{PQ}\|$  or  $W = \mathbf{F} \cdot \vec{PQ}$



| Work = Force Times Distance

$$= |\mathbf{F}| D$$

$$= \|(\text{proj}_{\vec{v}} \mathbf{F})\| \|\vec{v}\|$$

$$= \left\| \frac{\mathbf{F} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} \right\| \|\vec{v}\|$$

$$= \left\| \frac{\mathbf{F} \cdot \vec{v}}{\|\vec{v}\|^2} \right\| \|\vec{v}\| \|\vec{v}\|$$

$$= \|\mathbf{F} \cdot \vec{v}\| = |\mathbf{F} \cdot \vec{v}|$$

Some facts

$$\|\vec{7v}\| = 7\|\vec{v}\|$$

$$\underline{\mathbf{F}} \quad \|\vec{7v}\| = \|\langle 7v_1, 7v_2 \rangle\|$$

$$= \sqrt{(7v_1)^2 + (7v_2)^2}$$

$$= \sqrt{49v_1^2 + 49v_2^2}$$

$$= \sqrt{49(v_1^2 + v_2^2)}$$

$$= \sqrt{49} \sqrt{v_1^2 + v_2^2}$$

$$= 7\|\vec{v}\| \blacksquare$$

## 7. LarTrig9 3.4.007. (2446886)

$$\text{Find } \mathbf{u} \cdot \mathbf{v}. = (6)(-5) + (1)(4) = -30 + 4 = -26$$

$$\mathbf{u} = \langle 6, 1 \rangle$$

$$\mathbf{v} = \langle -5, 4 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} = \boxed{7a} \quad \boxed{-26}$$

**8. LarTrig9 3.4.008. (2447021)**

Find  $\mathbf{u} \cdot \mathbf{v}$ .  $= (3)(-3) + (8)(4) = -9 + 32 = 23$

$$\mathbf{u} = \langle 3, 8 \rangle$$

$$\mathbf{v} = \langle -3, 4 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} = 8a \quad \boxed{\text{ }} \quad \text{key } 23$$

**9. LarTrig9 3.4.011. (2447424)**

Find  $\mathbf{u} \cdot \mathbf{v}$ .  $= 6(1) + (-5)(-1) = 6 + 5 = 11$

$$\mathbf{u} = 6\mathbf{i} - 5\mathbf{j} = \langle 6, -5 \rangle$$

$$\mathbf{v} = \mathbf{i} - \mathbf{j} = \langle 1, -1 \rangle$$

$$\mathbf{u} \cdot \mathbf{v} = 9a \quad \boxed{\text{ }} \quad \text{key } 11$$

**10. LarTrig9 3.4.015. (2483705)**

Use the vector  $\mathbf{u} = \langle 3, 3 \rangle$  to find the indicated quantity.

$$\mathbf{u} \cdot \mathbf{u} = (3)(3) + (3)(3) = 18$$

$$\mathbf{u} \cdot \mathbf{u} = 10a \quad \boxed{18}$$

State whether the result is a vector or a scalar.

The result is a ---Select--- ▾ 10b  scalar .  
scalar

**11. LarTrig9 3.4.016. (2446422)**

Use the vectors  $\mathbf{u} = \langle 2, 4 \rangle$  and  $\mathbf{v} = \langle -5, 3 \rangle$  to find the indicated quantity.

$$3\mathbf{u} \cdot \mathbf{v} = 3(\mathbf{u} \cdot \mathbf{v}) = 3(2(-5) + (4)(3)) = 3(-10 + 12) = 6$$

$$3\mathbf{u} \cdot \mathbf{v} = 11a \quad \boxed{6}$$

$$3\mathbf{u} \cdot \mathbf{v} = \langle 6, 12 \rangle \cdot \langle -5, 3 \rangle \\ = -30 + 36 = 6$$

State whether the result is a vector or a scalar.

The result is a ---Select--- ▾ 11b  scalar .

**□ 12. LarTrig9 3.4.017. (2447112)**

Use the vectors  $\mathbf{u} = \langle 4, 4 \rangle$  and  $\mathbf{v} = \langle -5, 1 \rangle$  to find the indicated quantity.

$$(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = (4(-5) + 4(1)) \langle -5, 1 \rangle = (-20 + 4) \langle -5, 1 \rangle$$

$$(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = [12a] \boxed{\quad} \boxed{\langle 80, -16 \rangle} = -16 \langle -5, 1 \rangle = \langle 80, -16 \rangle$$

State whether the result is a vector or a scalar.

The result is a  **12b**  **vector**.

**□ 13. LarTrig9 3.4.018. (2548103)**

Use the vectors  $\mathbf{u} = \langle 2, 3 \rangle$ ,  $\mathbf{v} = \langle -4, 1 \rangle$ , and  $\mathbf{w} = \langle 2, -2 \rangle$  to find the indicated quantity.

$$(\mathbf{v} \cdot \mathbf{u})\mathbf{w} = (2(-4) + 3(1)) \mathbf{w} = -5 \langle 2, -2 \rangle = \langle -10, 10 \rangle$$

$$(\mathbf{v} \cdot \mathbf{u})\mathbf{w} = [13a] \boxed{\quad} \boxed{\langle -10, 10 \rangle}$$

State whether the result is a vector or a scalar.

The result is a  **13b**  **vector**.

## □ 14. LarTrig9 3.4.022. (2548343)

Use the vector  $\mathbf{u} = \langle 2, 2 \rangle$  to find the indicated quantity.

$$2 - \|\mathbf{u}\| = 2 - \sqrt{2^2 + 2^2} = 2 - \sqrt{8} = \boxed{2 - 2\sqrt{2}}$$

$$2 - \|\mathbf{u}\| = \boxed{14a} \quad \boxed{2 - 2\sqrt{2}}$$

State whether the result is a vector or a scalar.

The result is a   14b  scalar .

## □ 15. LarTrig9 3.4.023. (2446310)

Use the vectors  $\mathbf{u} = \langle 5, 5 \rangle$ ,  $\mathbf{v} = \langle -6, 4 \rangle$ , and  $\mathbf{w} = \langle 5, -1 \rangle$  to find the indicated quantity.

$$(\mathbf{u} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{w}) = 5(-6) + (5)(4) - (5(5) + (5)(-1)) = -10 - (20)$$

$$(\mathbf{u} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{w}) = \boxed{15a} \quad \boxed{-30} = -30$$

State whether the result is a vector or a scalar.

The result is a   15b  scalar .

$$= \overline{\mathbf{u}} \cdot (\overline{\mathbf{v}} - \overline{\mathbf{w}})$$

$$= \overline{\mathbf{u}} \cdot \langle -11, 5 \rangle$$

$$= \langle 5, 5 \rangle \cdot \langle -11, 5 \rangle$$

$$= -55 + 25 = -30$$

16. LarTrig9 3.4.024. (2446680)

Use the vectors  $\mathbf{u} = \langle 2, 3 \rangle$ ,  $\mathbf{v} = \langle -4, 1 \rangle$ , and  $\mathbf{w} = \langle 2, -2 \rangle$  to find the indicated quantity.

$$(\mathbf{v} \cdot \mathbf{u}) - (\mathbf{w} \cdot \mathbf{v}) = \overline{\mathbf{v}} \cdot \overline{\mathbf{u}} - \overline{\mathbf{v}} \cdot \overline{\mathbf{w}} = \overline{\mathbf{v}} \cdot (\overline{\mathbf{u}} - \overline{\mathbf{w}}) = \overline{\mathbf{v}} \cdot \langle 0, 5 \rangle$$

$$(\mathbf{v} \cdot \mathbf{u}) - (\mathbf{w} \cdot \mathbf{v}) = 16a \quad \boxed{5} \quad = \langle -4, 1 \rangle \cdot \langle 0, 5 \rangle = 0 + 5 = 5$$

State whether the result is a vector or a scalar.

The result is a  16b

17. LarTrig9 3.4.025. (2446554)

Use the dot product to find the magnitude of  $\mathbf{u}$ .

$$\mathbf{u} = \langle -3, 4 \rangle \implies \|\mathbf{u}\| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\|\mathbf{u}\| = 17a \quad \boxed{5} \quad \|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} =$$

## □ 18. LarTrig9 3.4.026. (2558822)

Use the dot product to find the magnitude of  $\mathbf{u}$ .

$$\mathbf{u} = \langle 3, -5 \rangle \Rightarrow \|\mathbf{u}\| = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

$$\|\mathbf{u}\| = 18a \quad \boxed{\sqrt{34}}$$

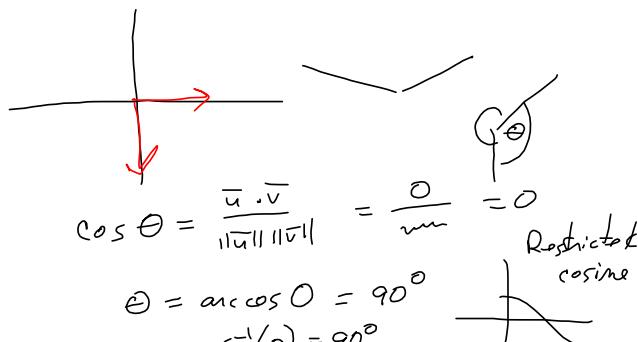
## □ 19. LarTrig9 3.4.031. (2446594)

Find the angle  $\theta$  between the vectors.

$$\mathbf{u} = \langle 4, 0 \rangle \quad \theta = 90^\circ$$

$$\mathbf{v} = \langle 0, -5 \rangle$$

$$\theta = 19a \quad \boxed{90}^\circ$$



Calculator's  
 $\cos^{-1}$  key works with  
no problem, because  
 $0 \leq \theta \leq 180^\circ$

## □ 20. LarTrig9 3.4.032. (2446203)

Find the angle  $\theta$  between the vectors. (Round your answer to two decimal places.)

$$\mathbf{u} = \langle 3, 5 \rangle$$

$$\mathbf{v} = \langle 7, 0 \rangle$$

$$\theta = \boxed{20a} \quad \boxed{59.04}^\circ$$

$$\overline{\mathbf{u}} \cdot \overline{\mathbf{v}} = 21 + 0 = 21$$

$$\|\mathbf{u}\| = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$$

$$\|\mathbf{v}\| = 7$$

$$\cos \theta = \frac{\overline{\mathbf{u}} \cdot \overline{\mathbf{v}}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{21}{7\sqrt{34}} = \cos \theta$$

$$59.04^\circ \quad \approx 59.03624344 \approx \arccos\left(\frac{21}{7\sqrt{34}}\right) = \theta$$

## □ 21. LarTrig9 3.4.038. (2447192)

Find the angle  $\theta$  between the vectors. (Round your answer to two decimal places.)

$$\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} = \langle 2, -4 \rangle$$

$$\mathbf{v} = 5\mathbf{i} + 4\mathbf{j} = \langle 5, 4 \rangle$$

$$\theta = \boxed{21a} \quad \boxed{102.09}^\circ$$

$$\begin{aligned} -6/(2*\sqrt{5*41}) \\ -2095290887 \\ \cos^{-1}(\text{Ans}) \\ 102.0947571 \end{aligned}$$

$$\overline{\mathbf{u}} \cdot \overline{\mathbf{v}} = 10 - 16 = -6$$

$$\begin{aligned} \|\mathbf{u}\| &= \sqrt{2^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5} \\ \|\mathbf{v}\| &= \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41} \end{aligned}$$

$$\cos \theta = \frac{\overline{\mathbf{u}} \cdot \overline{\mathbf{v}}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-6}{2\sqrt{5}\sqrt{41}}$$

$$\begin{array}{r} 2 \\ | \\ 20 \\ 2 \\ | \\ 10 \\ 5 \end{array}$$

$$\begin{array}{l} \approx 102.09^\circ \\ \approx \theta \end{array}$$

## 22. LarTrig9 3.4.039. (2446737)

Find the angle  $\theta$  between the vectors. (Enter your answer in radians.)

$$\mathbf{u} = \cos\left(\frac{\pi}{5}\right)\mathbf{i} + \sin\left(\frac{\pi}{5}\right)\mathbf{j}$$

$$\mathbf{v} = \cos\left(\frac{5\pi}{6}\right)\mathbf{i} + \sin\left(\frac{5\pi}{6}\right)\mathbf{j}$$

$\theta =$

$$\frac{19\pi}{30}$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \cos \frac{\pi}{5} \cos \frac{5\pi}{6} + \sin \frac{\pi}{5} \sin \frac{5\pi}{6}$$

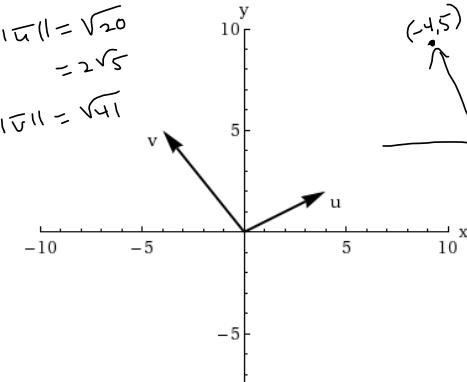
$$\begin{aligned} & 102.0947571 \\ & \cos^{-1}(\cos(\pi/5)\cos(5\pi/6) + \sin(\pi/5)\sin(5\pi/6)) \\ & 1.989675347 \\ & \text{Ans} * 180/\pi \\ & 114 \end{aligned}$$

## 23. LarTrig9 3.4.042. (2446386)

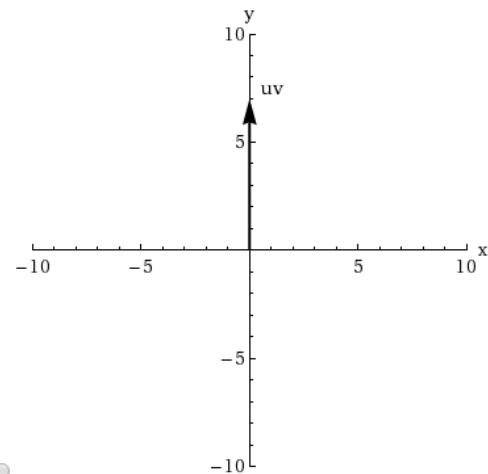
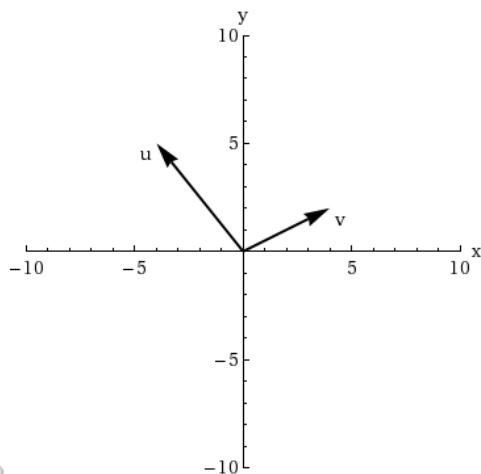
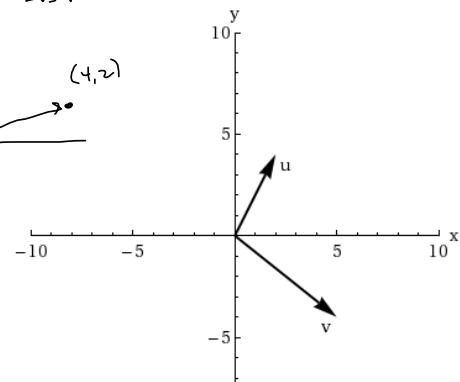
Graph the vectors and find the degree measure of the angle  $\theta$  between the vectors. (Round your answer to two decimal places.)

$$\begin{aligned} \mathbf{u} &= 4\mathbf{i} + 2\mathbf{j} \\ \mathbf{v} &= -4\mathbf{i} + 5\mathbf{j} \end{aligned}$$

$$\begin{aligned} \|\mathbf{u}\| &= \sqrt{20} \\ &= 2\sqrt{5} \\ \|\mathbf{v}\| &= \sqrt{41} \end{aligned}$$



$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-6}{2\sqrt{5}\sqrt{41}} \Rightarrow \theta \approx 102.09^\circ \quad \text{See prev. work!}$$



23a

## □ 24. LarTrig9 3.4.045. (2446786)

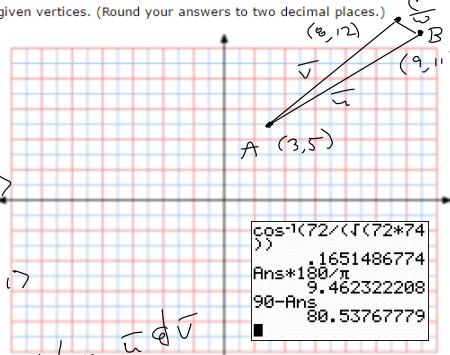
Use vectors to find the interior angles of the triangle with the given vertices. (Round your answers to two decimal places.)  
 (3, 5), (9, 11), (8, 12)

24a [ ] 9.46 ° (smallest value)  
 24b [ ] 80.54 °  
 24c [ ] 90 ° (largest value)

$$\overrightarrow{AB} = \vec{u} = \langle 9-3, 11-5 \rangle = \langle 6, 6 \rangle$$

$$\overrightarrow{AC} = \vec{v} = \langle 8-3, 12-5 \rangle = \langle 5, 7 \rangle$$

$$\overrightarrow{BC} = \vec{w} = \langle 8-9, 12-11 \rangle = \langle -1, 1 \rangle$$



$$\cos^{-1}(72/\sqrt{72*74}) \\ \text{Ans} * 180/\pi \\ 9.462322208 \\ 90 - \text{Ans} \\ 80.53767779$$

$\vec{u} \perp \vec{w}$

$180^\circ - \theta$ , where  
 $\cos \theta = \frac{\vec{u} \cdot \vec{w}}{\|\vec{u}\| \|\vec{w}\|} = \frac{0}{\sqrt{72} \sqrt{74}} = 0 \Rightarrow \theta = 90^\circ$

Angle between  $\vec{u}$  &  $\vec{v}$

$$\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \cos A = \frac{70+42}{\sqrt{6^2+6^2} \sqrt{5^2+7^2}}$$

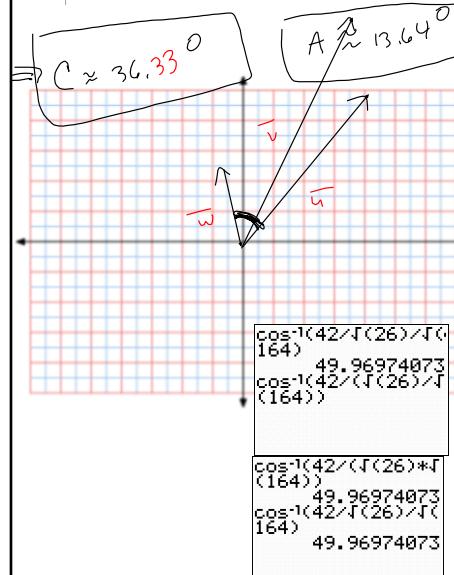
$$= \frac{72}{\sqrt{72} \sqrt{74}} \quad 80.54^\circ$$

$9.46^\circ$

## □ 25. LarTrig9 3.4.046. (2548051)

Use vectors to find the interior angles of the triangle with the given vertices. (Round your answers to two decimal places.)  
 (-3, -7), (4, 8), (5, 3)

- 25a [ ] 13.64 ° (smallest value)  
 25b [ ] 36.33 °  
 25c [ ] 130.03 ° (largest value)

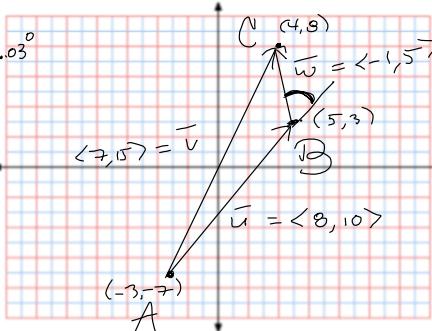


$$\cos^{-1}(42/\sqrt{26})/\sqrt{164} \\ 49.96974073 \\ \cos^{-1}(42/(\sqrt{26})/\sqrt{164}) \\ \cos^{-1}(42/(\sqrt{26})/\sqrt{164}) \\ 49.96974073 \\ \cos^{-1}(42/(\sqrt{26})/\sqrt{164}) \\ 49.96974073$$

$$\begin{matrix} 149 \\ 25 \\ 234 \end{matrix}$$

$$206/\sqrt{274}/\sqrt{164} \\ 9717846055 \\ \cos^{-1}(\text{Ans}) \\ 13.64291478$$

$$\begin{matrix} .9717846055 \\ \cos^{-1}(\text{Ans}) \\ 13.64291478 \\ \text{Ans} + 130.0302579 - \\ 180 \\ -36.32682732 \end{matrix}$$



$$= \frac{42}{\sqrt{26} \sqrt{164}} \\ \Rightarrow \theta \approx 49.96974073^\circ \\ \text{So angle } B \approx 180^\circ - 49.96974073^\circ \\ \approx 130.0302597^\circ \\ \approx 130.03^\circ \approx B$$

$$\vec{v} = \langle 7, 15 \rangle$$

$$\vec{u} = \langle 8, 10 \rangle$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{206}{\sqrt{274} \sqrt{164}} \Rightarrow$$

$$\theta \approx 13.64291478$$

$A \approx 13.64^\circ$

$\Rightarrow C \approx 36.33^\circ$

## □ 26. LarTrig9 3.4.049. (2446940)

Find  $\mathbf{u} \cdot \mathbf{v}$ , where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\|\mathbf{u}\| = 5, \|\mathbf{v}\| = 6, \theta = \frac{2\pi}{3}$$

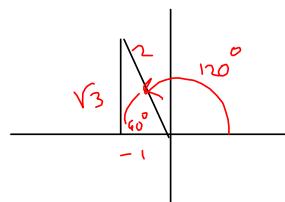
26a

-15

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{\left(\frac{2\pi}{3}\right)\left(\frac{180}{\pi}\right)}{(5)(6)} = 120^\circ$$

$$\cos\left(\frac{2\pi}{3}\right) = \frac{\mathbf{u} \cdot \mathbf{v}}{(5)(6)} \Rightarrow$$

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= 30 \cos\left(\frac{2\pi}{3}\right) \\ &= 30 \cos(120^\circ) \\ &= 30\left(-\frac{1}{2}\right) = -15 = \mathbf{u} \cdot \mathbf{v} \end{aligned}$$



## □ 27. LarTrig9 3.4.050. (2548114)

Find  $\mathbf{u} \cdot \mathbf{v}$ , where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

$$\|\mathbf{u}\| = 60, \|\mathbf{v}\| = 240, \theta = \frac{\pi}{6}$$

27a

7200 $\sqrt{3}$ 

Same exact deal.

Different #s.

check your answer!

## 28. LarTrig9 3.4.058. (2446200)

Determine whether  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal.

$$\mathbf{u} = \langle 2 \cos \theta, 2 \sin \theta \rangle$$

$$\mathbf{v} = \langle 2 \sin \theta, -2 \cos \theta \rangle$$

Looking for  $\overline{\mathbf{u}} \cdot \overline{\mathbf{v}} = 0$  for orthogonal

orthogonal

not orthogonal

$$\overline{\mathbf{u}} \cdot \overline{\mathbf{v}} = 4 \sin \theta \cos \theta - 4 \sin \theta \cos \theta = 0$$

## 29. LarTrig9 3.4.059. (2447118)

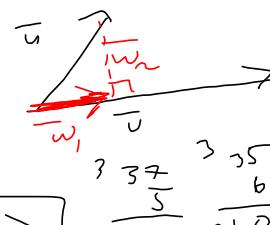
Find the projection of  $\mathbf{u}$  onto  $\mathbf{v}$ .

$$\mathbf{u} = \langle 5, 5 \rangle$$

$$\mathbf{v} = \langle 6, 1 \rangle$$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \boxed{29a} \quad \left\langle \frac{210}{37}, \frac{35}{37} \right\rangle$$

$$\overline{\mathbf{w}}_1 = \text{proj}_{\mathbf{v}} \overline{\mathbf{u}} = \frac{\overline{\mathbf{u}} \cdot \overline{\mathbf{v}}}{\|\overline{\mathbf{v}}\|^2} \overline{\mathbf{v}} = \frac{\frac{35}{37} \overline{\mathbf{v}}}{\frac{35}{37}} \overline{\mathbf{v}} = \frac{35}{37} \langle 6, 1 \rangle$$



$$\overline{\mathbf{w}}_2 = \overline{\mathbf{u}} - \overline{\mathbf{w}}_1 = \langle 5, 5 \rangle - \frac{35}{37} \langle 6, 1 \rangle = \left\langle \frac{185}{37}, \frac{185}{37} \right\rangle - \left\langle \frac{210}{37}, \frac{35}{37} \right\rangle$$

$$= \left\langle \frac{-25}{37}, \frac{150}{37} \right\rangle = \overline{\mathbf{w}}_2$$

Write  $\mathbf{u}$  as the sum of two orthogonal vectors, one of which is  $\text{proj}_{\mathbf{v}} \mathbf{u}$ .

$$\mathbf{u} = \text{proj}_{\mathbf{v}} \mathbf{u} + \boxed{29b} \quad \left\langle -\frac{25}{37}, \frac{150}{37} \right\rangle$$

$$\begin{matrix} 185 \\ -210 \\ -25 \end{matrix}$$

$$\overline{\mathbf{u}} = \overline{\mathbf{w}}_1 + \overline{\mathbf{w}}_2$$

$$= \text{proj}_{\mathbf{v}} \overline{\mathbf{u}} + \left\langle -\frac{25}{37}, \frac{150}{37} \right\rangle$$

$$= \frac{1}{37} \left( \langle 185, 185 \rangle + \langle -25, 150 \rangle \right)$$

## □ 30. LarTrig9 3.4.060. (2456374)

Find the projection of  $\mathbf{u}$  onto  $\mathbf{v}$ .

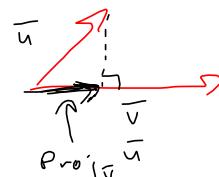
$$\mathbf{u} = \langle 6, 4 \rangle$$

$$\mathbf{v} = \langle 6, -9 \rangle$$

$$\text{proj}_{\mathbf{v}} \mathbf{u} = \boxed{30a} \quad \boxed{\langle 0, 0 \rangle}$$

$$\text{proj}_{\mathbf{v}} \bar{\mathbf{u}} = \frac{\bar{\mathbf{u}} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} \mathbf{v}$$

$$= (\cos \theta) \left( \|\bar{\mathbf{u}}\| \right) \frac{\mathbf{v}}{\|\mathbf{v}\|}$$



Write  $\mathbf{u}$  as the sum of two orthogonal vectors, one of which is  $\text{proj}_{\mathbf{v}} \mathbf{u}$ .

$$\mathbf{u} = \text{proj}_{\mathbf{v}} \mathbf{u} + \boxed{30b} \quad \boxed{\langle 6, 4 \rangle}$$

$$= \frac{\bar{\mathbf{u}} \cdot \mathbf{v}}{\|\bar{\mathbf{u}}\| \|\mathbf{v}\|} \|\bar{\mathbf{u}}\| \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

$$\bar{\mathbf{u}} \cdot \mathbf{v} = (6)(6) + (4)(-9) = 0 !$$

$$\Rightarrow \text{proj}_{\mathbf{v}} \bar{\mathbf{u}} = \langle 0, 0 \rangle$$

$$0 \mathbf{v} = 0 \langle 6, -9 \rangle$$

$$= \langle 6, 4 \rangle + \langle 0, 0 \rangle = \langle 6, 4 \rangle$$

$$= \langle 0, 0 \rangle = \bar{\mathbf{u}}$$

