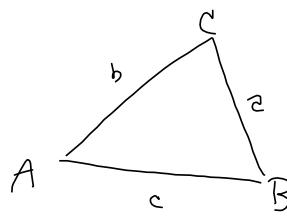


S 3.2 Law of Cosines



$$\begin{array}{l} \text{SSS} \\ c^2 = a^2 + b^2 - 2ab \cos C \\ a^2 = b^2 + c^2 - 2bc \cos A \\ b^2 = a^2 + c^2 - 2ac \cos B \end{array}$$

Heron's Formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$

1. + -/1 points LarTrig9 3.2.001.

Fill in the blank.

When you are given three sides of a triangle, you use the Law of to find the three angles of the triangle.

Cosines SSS

2. + -/1 points LarTrig9 3.2.002.

Fill in the blank.

When you are given two angles and any side of a triangle, you use the Law of to solve the triangle.

Sines AAS, ASA

3. + -/1 points LarTrig9 3.2.004.

Fill in the blank.

The Law of Cosines can be used to establish a formula for finding the area of a triangle called Formula.

Heron's

4. + -/3 points LarTrig9 3.2.005.

Use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

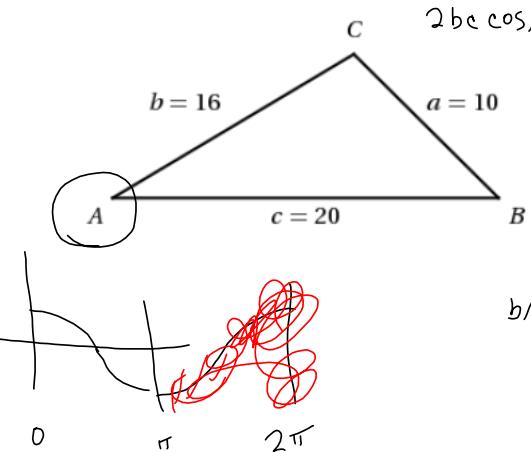
$$A = \boxed{\quad}^\circ$$

sss switch

$$B = \boxed{\quad}^\circ$$

$$C = \boxed{\quad}^\circ$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$



$$2bc \cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{16^2 + 20^2 - 10^2}{2(16)(20)} = \frac{139}{160} = .86875$$

$$\cos^{-1}(\cos A) = A = \cos^{-1}(.86875)$$

$$\text{b/c } 0 < A < 180^\circ = \pi \approx 29.68629523^\circ$$

$$\approx 29.69^\circ \approx A$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{10^2 + 20^2 - 16^2}{2(10)(20)} = \frac{100 + 400 - 256}{400}$$

$$= \frac{244}{400} = \frac{122}{200} = \frac{61}{100} = .61$$

$$\Rightarrow B = \arccos(.61) \approx 52.41047904^\circ$$

$$\approx 52.41^\circ$$

$\cos^{-1}(\text{Ans})$	29.68629523°	$\approx A$
$(10^2 + 20^2 - 16^2) / (2 * 10 * 20)$.61	
$\cos^{-1}(\text{Ans})$	52.41049704°	$\approx B$

$$C \approx 180 - 52.41 - 29.69^\circ$$

$$= 97.90^\circ \approx C$$

5. + -3 points LarTrig9 3.2.010.MI.

Use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

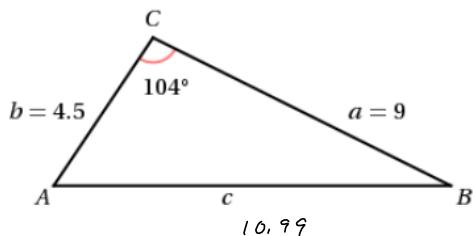
$$A = \boxed{\quad}^\circ$$

$$B = \boxed{\quad}^\circ$$

$$c = \boxed{\quad}$$

SAS

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4.5^2 + 10.99^2 - 9^2}{2(4.5)(10.99)} = \text{oops! need } c, 10.99$$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{4.5^2 + 10.99^2 - 9^2}{2(4.5)(10.99)}$$

$$\approx .607414485$$

$$A \approx \arccos(.607414485)$$

$$\approx 52.59721273^\circ \approx \boxed{52.60^\circ \approx A}$$

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos C \\ &= 9^2 + 4.5^2 - 2(9)(4.5) \cos(104^\circ) \\ &\approx 120.8456735 \end{aligned}$$

$$\Rightarrow c \approx \sqrt{120.8456735} \approx 10.9930 \quad \boxed{c \approx 10.99}$$

$$\begin{aligned} &(4.5^2 + \text{Ans}^2 - 9^2) / (2 * 4.5 * \text{Ans}) \\ &.607414485 \\ &\cos^{-1}(\text{Ans}) \\ &52.59721273 \\ &180 - \text{Ans} - 104 \\ &23.40278727 \quad \blacksquare \end{aligned}$$

$$\approx \boxed{B \approx 23.40^\circ}$$

6. +/-3 points LarTrig9 3.2.014.

Use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

$$a = 50, b = 27, c = 70$$

if

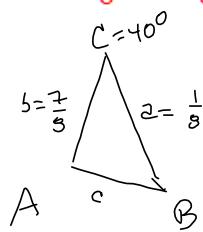
SSS. See #4 (#5 in Book)

7. +/-3 points LarTrig9 3.2.023.

Use the Law of Cosines to solve the triangle. Round your answers to two decimal places.

$$C = 40^\circ, a = \frac{1}{8}, b = \frac{7}{8}$$

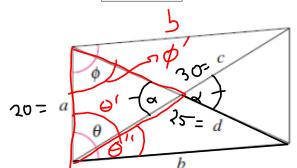
SAS (#5 / #10 in text)



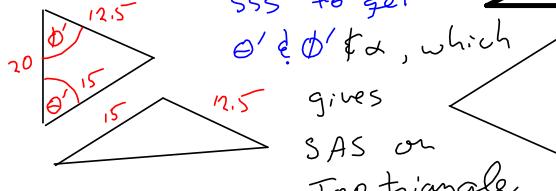
8. -3 points LarTrig9 3.2.029.

Complete the table by solving the parallelogram shown in the figure. (The lengths of the diagonals are given by c and d . Round your answers to two decimal places.)

a	b	c	d	θ	ϕ
20		30	25		



Solving, finally,
for ϕ & θ to
get $\theta = \theta' + \theta''$
 $\phi = \phi' + \phi''$



Diagonals of parallelogram

bisect each other

sss to get

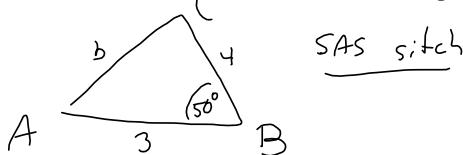
 $\theta' \& \phi' \& \phi'',$ which
gives
SAS on
Top triangle.

9. -4 points LarTrig9 3.2.031.

Determine whether the Law of Sines or the Law of Cosines is needed to solve the triangle.

$$a = 4, c = 3, B = 50^\circ$$

- Law of Sines
- Law of Cosines



Solve (if possible) the triangle. If two solutions exist, find both. Round your answers to

$$A = 82.03^\circ \quad \text{two decimal places. (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)}$$

$$C = 47.97^\circ$$

$$b = 3.09$$

A little sketch
on rounding,

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos B \\ &= 3^2 + 4^2 - 2(3)(4) \cos(50^\circ) \\ &\approx 9.573097368 \\ &= 47.97 \end{aligned}$$

$$\Rightarrow b \approx \pm 3.094042238$$

SAS sketch

$$\begin{aligned} 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos(50^\circ) &= 9.573097368 \\ \text{Ans}^{\wedge}.5 &= 3.094042238 \\ \blacksquare \end{aligned}$$

$$\text{so } b \approx 3.09$$

$$\begin{aligned} \frac{\sin A}{4} &\approx \frac{\sin 50^\circ}{3.09...} \\ \sin A &\approx \frac{4 \sin 50^\circ}{3.09...} \\ &\approx .9963... \end{aligned}$$

$$\begin{aligned} \text{Ans}^{\wedge}.5 &= 3.094042238 \\ 4 \sin(50^\circ) / \text{Ans} &= 9903477513 \\ \sin^{-1}(\text{Ans}) &= 82.0328714 \\ \blacksquare \end{aligned}$$

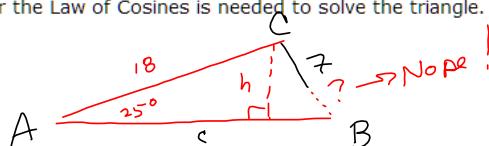
$$\begin{aligned} A &\approx \sin^{-1}(0.9903477513) \\ &\approx 82.03^\circ \end{aligned}$$

10. + -4 points LarTrig9 3.2.033.

Determine whether the Law of Sines or the Law of Cosines is needed to solve the triangle.

$$A = 25^\circ, a = 7, b = 18$$

- Law of Sines
- Law of Cosines



Solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

$$B = \boxed{\text{NoPE}}^\circ$$

$$C = \boxed{\text{NoPE}}^\circ$$

$$c = \boxed{\text{NoPE}}$$

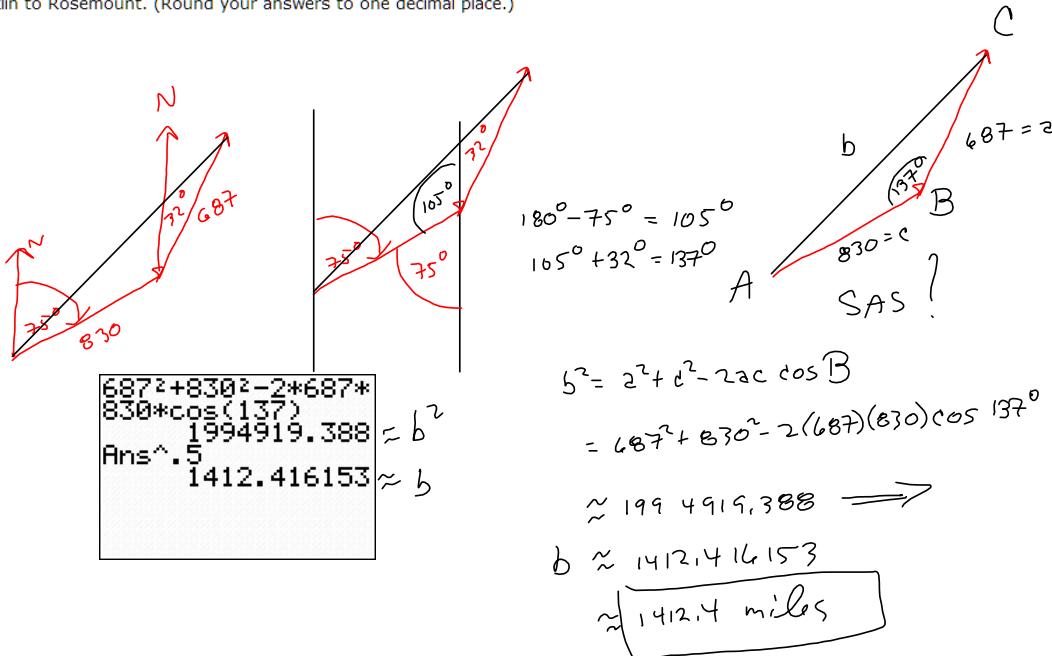
See; if $a=7$ is long enough:

$$\frac{h}{18} = \sin 25^\circ$$

$$h = 18 \sin 25^\circ \approx 7.607 > 7$$

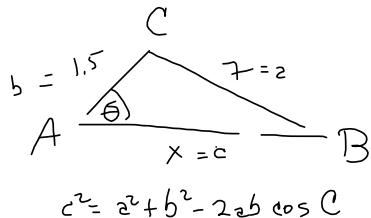
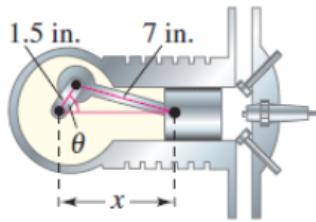
side $a=7$ can't reach!

11. + -2 points LarTrig9 3.2.046.

A plane flies 830 miles from Franklin to Centerville with a bearing of 75° . Then it flies 687 miles from Centerville toRosemount with a bearing of 32° . Draw a figure that visually represents the situation, and find the straight-line distance and bearing from Franklin to Rosemount. (Round your answers to one decimal place.)

12. +4 points LarTrig9 3.2.056.

An engine has a seven-inch connecting rod fastened to a crank (see figure).



- Use the Law of Cosines to write an equation giving the relationship between x and θ .
- Write x as a function of θ . (Select the sign that yields positive values of x .)
- Use a graphing utility to graph the function in part (b).
- Use the graph in part (c) to determine the maximum value of x .

$$a^2 = b^2 + c^2 - 2ac \cos A$$

$$7^2 = 1.5^2 + x^2 - 2(7)x \cos \theta$$

$$49 = 2.25 + x^2 - 14x \cos \theta \quad \text{a}$$

$$x^2 - 14x \cos \theta + 2.25 = 49$$

$$x^2 (14 \cos \theta) - 46.75 = 0$$

$$\begin{array}{r} 49 \\ - 2.25 \\ \hline 46.75 \end{array}$$

$$b^2 = 4ac$$

$$(14 \cos \theta)^2 - 4(1)(-46.75)$$

$$\begin{array}{r} 196 \\ 187 \\ \hline 383 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{14 \cos \theta \pm \sqrt{196 \cos^2 \theta + 187}}{2(1)}$$

$$\begin{aligned} &= 196 \cos^2 \theta + 187 \\ &0 \leq \cos^2 \theta \leq 1 \\ &0 + 187 \leq 196 \cos^2 \theta + 187 \leq 196 + 187 \\ &187 \leq \sqrt{196 \cos^2 \theta + 187} \leq \sqrt{383} \end{aligned}$$

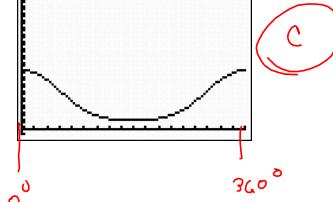
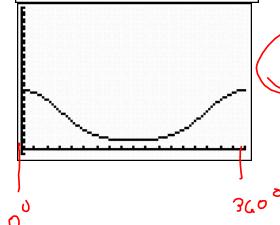
$$\begin{aligned} &\text{Keep } ' + \text{ it Positive} \\ &x = \frac{14 \cos \theta + \sqrt{196 \cos^2 \theta + 187}}{2} \quad \text{b} \end{aligned}$$

1412.416153
4*46.75
187
196
$y_1(0)$
16.7851929

d

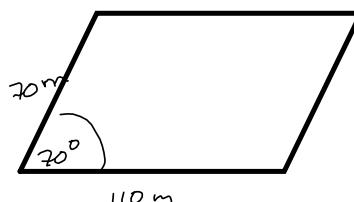
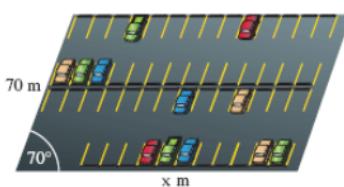
$\approx \text{MAX of } x$

```
Plot1 Plot2 Plot3
Y1: 14cos(X)+sqrt(196cos(X)^2+187)
/2
Y2:
Y3:
Y4:
Y5:
```

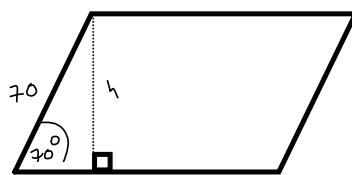
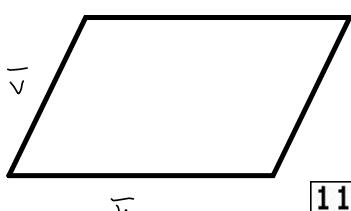


13. +1 points LarTrig9 3.2.058.

A parking lot has the shape of a parallelogram (see figure). The lengths of two adjacent sides are 70 meters and $x = 110$ meters. The angle between the two sides is 70° . What is the area of the parking lot? (Round your answer to one decimal place.)



$$\text{Area} = \text{Base} \cdot \text{Height}$$



$$\frac{h}{70} = \sin 70^\circ$$

$$h = 70 \sin 70^\circ$$

$$\text{Area} = 110 \cdot 70 \sin 70^\circ$$

$$\text{Area} = \| \vec{u} \times \vec{v} \|$$

\vec{u} = magnitude of
the cross product.

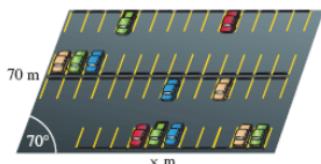
$$110 \cdot 70 \sin(70) \\ 7235.63318$$

$$\approx \text{Area} \approx 7235.6 \text{ m}^2$$

#58 from textbook

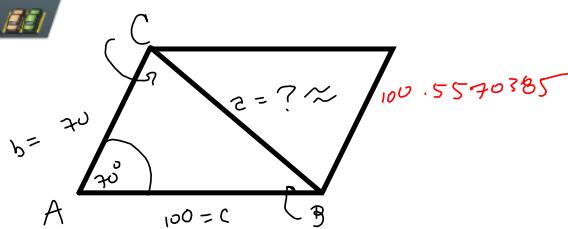
13. +1 points LarTrig9 3.2.058.

A parking lot has the shape of a parallelogram (see figure). The lengths of two adjacent sides are 70 meters and $x = 110$ meters. The angle between the two sides is 70° . What is the area of the parking lot? (Round your answer to one decimal place.)



$$\text{ANS: } A \approx 6577.8 \text{ m}^2$$

$x = 100$ in
book



$$s = \frac{a+b+c}{2}$$

$$A_{\text{triangle}} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\begin{aligned} & 70^2 + 100^2 - 2 \cdot 70 \cdot 10 \\ & 0 \cdot \cos(70) \\ & 10111.71799 \\ & \text{Ans}^{.5} \\ & 100.5570385 \\ & \blacksquare \end{aligned}$$

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= 70^2 + 100^2 - 2(70)(100) \cos(70^\circ) \\ &\approx 10111.71799 \\ &\rightarrow a \approx 100.5570385 \end{aligned}$$

$$\begin{aligned} & \text{Ans} * 2 \\ & 13155.69672 \\ & 70^2 + 100^2 - 2 \cdot 70 \cdot 10 \\ & 0 \cdot \cos(70) \\ & 10111.71799 \\ & \text{Ans}^{.5} \\ & 100.5570385 \end{aligned}$$

≈ 2

Herons:

$$\begin{aligned} & 100.5570385 \\ & \sqrt{(135.2785193 * (135.2785193 - \text{Ans}) * (135.2785193 - 70) * (135.2785193 - 100))} \\ & 3288.924179 \end{aligned}$$

\approx Area of
the triangle

So area of parallelogram is
 $2 * \text{previous}$

$$\begin{aligned} & 35.2785193 - \text{Ans}) * (135.2785193 - 70) * (135.2785193 - 100)) \\ & 3288.924179 \\ & 2 * \text{Ans} \\ & 6577.848358 \end{aligned}$$

\approx Area of
parallelogram