

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

SAS? Not so much

See "Law of Cosines"

AAS, ASA always good

ASS not so much. 3 possibilities:

- ① one soln
- ② two solns
- ③ No solns

Assume it's angle A that we're given,  
and two sides given

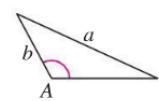
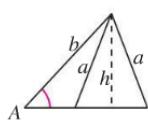
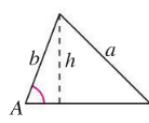
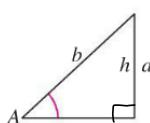
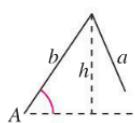
Let  $h = b \sin A$ . This is the altitude of the triangle.

(from  $\frac{h}{b} = \sin A$ )

$A$  is acute.

$A$  is obtuse.

Sketch



Necessary condition

$a < h$

Triangles possible

None

$a = h$

One

$a \geq b$

One

$h < a < b$

Two

$a \leq b$

None

$a > b$

One

1.  -/1 points LarTrig9 3.1.001.

Fill in the blank.

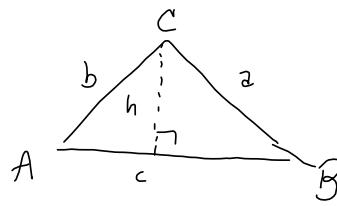
$A(n)$   triangle is a triangle that has no right angle.

obtuse

2.  -/1 points LarTrig9 3.1.004.

Fill in the blank.

The area of an oblique triangle is given by  $\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \underline{\underline{\frac{1}{2}ac \sin B}}$ .



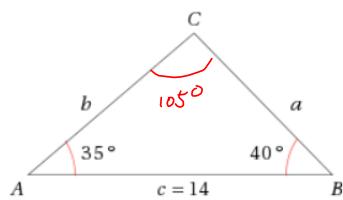
$$\begin{aligned} \text{Area} &= \frac{1}{2}(\text{base})(\text{height}) \\ &= \frac{1}{2}c \cdot b \sin A \\ &= \frac{1}{2}bc \sin A \end{aligned}$$

$$\frac{h}{b} = \sin A$$

$$h = b \sin A$$

3. +3 points LarTrig9 3.1.006.

Use the Law of Sines to solve the triangle. Round your answers to two decimal places.



$$\begin{aligned} C &= 180 - 40 - 35 \\ &= 180 - 75 = 105^\circ = C \end{aligned}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \implies \frac{a}{\sin 35^\circ} = \frac{14}{\sin 105^\circ}$$

$$\implies a = \frac{14 \sin 35^\circ}{\sin 105^\circ} \approx 8.31 \approx a$$

$$\therefore b = \frac{14 \sin 40^\circ}{\sin 105^\circ} \approx 9.32 \approx b$$

4. +/-3 points LarTrig9 3.1.011.

Use the Law of Sines to solve the triangle. Round your answers to two decimal places.

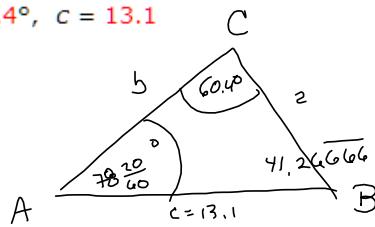
$$A = 78^\circ 20', C = 60.4^\circ, c = 13.1$$

$$B \approx 41.3^\circ$$

$$B = \boxed{\quad}^\circ$$

$$a = \boxed{\quad}$$

$$b = \boxed{\quad}$$



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$.26 = 1 - \frac{4}{10}$$

$$\frac{a}{\sin(78^\circ 20')} = \frac{c}{\sin(60.4^\circ)}$$

$$a = \frac{13.1 \sin(78^\circ 20')}{\sin(60.4^\circ)}$$

$$\approx 14.8 \approx a$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b = \frac{c \sin B}{\sin C} = \frac{13.1 \sin(41.26)}{\sin(60.4^\circ)} \approx \boxed{10.0 \approx b}$$

$$\begin{aligned} & -41.26666667 \\ & 13.1 \sin(41.6666666667) / \sin(60.4) \\ & 10.01595932 \\ & 13.1 \sin(78+20/60) / \sin(60.4) \\ & 14.75495913 \end{aligned}$$

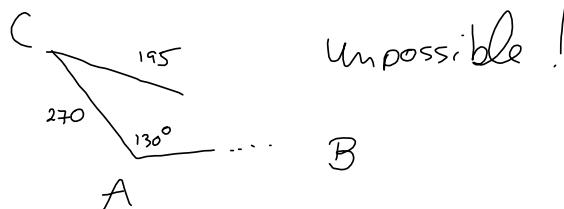
5. +/-3 points LarTrig9 3.1.026.MI.

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal place:

$$A = 130^\circ, a = 195, b = 270$$

(If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)

$$\begin{aligned} B &= \boxed{\quad}^\circ \\ C &= \boxed{\quad}^\circ \\ c &= \boxed{\quad} \end{aligned}$$



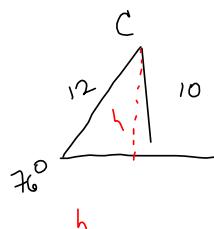
Impossible!

## 6. + -3 points LarTrig9 3.1.027.

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to two decimal places.

$A = 76^\circ$ ,  $a = 10$ ,  $b = 12$  (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)

$$\begin{array}{l} B = \boxed{\phantom{00}}^\circ \\ C = \boxed{\phantom{00}}^\circ \\ c = \boxed{\phantom{00}} \end{array}$$



$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$b = a \sin B \approx 11.64$$

So  $b > 10$ , so side A  
can't reach side c

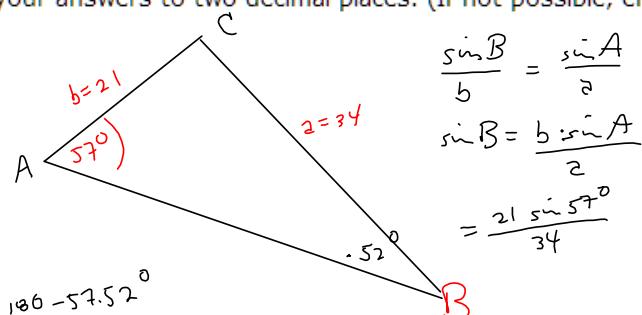
Unpossible

## 7. + -3 points LarTrig9 3.1.028.

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both.

$A = 57^\circ$ ,  $a = 34$ ,  $b = 21$

$$\begin{array}{l} B = \boxed{\phantom{00}}^\circ \\ C = \boxed{\phantom{00}}^\circ \\ c = \boxed{\phantom{00}} \end{array}$$



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{b \sin A}{a}$$

$$= \frac{21 \sin 57^\circ}{34}$$

$21 * \sin(57^\circ) / 34$	$\approx B$
$.5180024096$	
$180 - \text{Ans}_1$	$\approx C$
$122.4819976$	
$34 \sin(\text{Ans}) / \sin(57^\circ)$	$\approx c$
$34.19822839$	

$$\frac{c}{\sin C} = \frac{a}{\sin A} \Rightarrow c = \frac{a \sin C}{\sin A}$$

$$= \frac{34 \sin(122.4819976^\circ)}{\sin(57^\circ)}$$

No. This is  $\sin(B)$ . We want B. Neglected to take inverse sine of this, to find the real B.  
See  
3-1-notes-updated.pdf.

8. + -6 points LarTrig9 3.1.029.

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both.

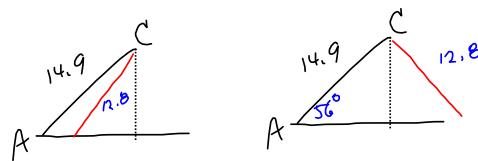
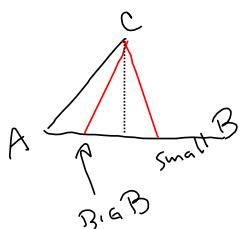
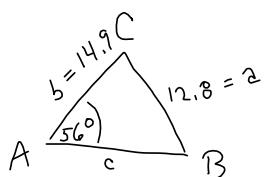
 $A = 56^\circ, a = 12.8, b = 14.9$  Round your answers to two decimal places.

Case 1:

$$\begin{array}{l} B = \boxed{\quad}^\circ \text{ (smaller } B\text{-value)} \\ C = \boxed{\quad}^\circ \\ c = \boxed{\quad} \end{array}$$

Case 2: (If a triangle is not possible, enter IMPOSSIBLE in each corresponding answer blank.)

$$\begin{array}{l} B = \boxed{\quad}^\circ \text{ (larger } B\text{-value)} \\ C = \boxed{\quad}^\circ \\ c = \boxed{\quad} \end{array}$$

Let's see if  $a$  is longer than the height

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = \frac{b \sin A}{a}$$

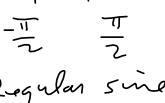
$$\begin{array}{l} \frac{h}{14.9} = \sin A \\ h = 14.9 \sin A \\ = 14.9 \sin 56^\circ \approx 12.4 < 12.8 = a, \text{ so 2 solns} \end{array}$$

Restricted sine for  
 $\sin^{-1}(x) = \arcsin(x)$ 

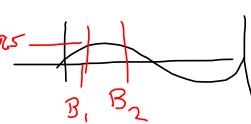
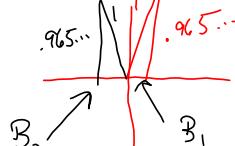
$$\sin B = \frac{14.9 \sin(56^\circ)}{12.8} \approx .9650515493$$

$$B_1 \approx \sin^{-1}(0.9650515493)$$

$$\boxed{B_1 \approx 74.81^\circ}$$



Regular sine:



From CALCULATOR

$$B_2 = 180^\circ - B_1$$

$$\boxed{B_2 \approx 105.19^\circ \approx B}$$

$$\begin{array}{l} 12.35265983 \\ \text{Ans}/12.8 \\ .9650515493 \\ \sin^{-1}(\text{Ans}) \\ 74.80766236 \\ \text{Ans}-180 \\ -105.1923376 \end{array}$$

9. +3 points LarTrig9 3.1.031.

Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both.

$$A = 101^\circ, a = b = 28$$

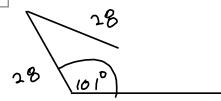
$$B = \boxed{\quad}$$

Round your answers to two decimal places. (If a triangle is not possible,

$$C = \boxed{\quad}$$

enter IMPOSSIBLE in each corresponding answer blank.)

$$c = \boxed{\quad}$$



Degenerate (No) triangle.

If  $A$  were acute, this would  
be isosceles



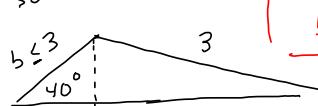
10. + 1/3 points LarTrig9 3.1.035.

Find values for  $b$  such that the triangle has one solution, two solutions, and no solution.

$$A = 40^\circ, a = 3$$

- (a) one solution
- (b) two solutions
- (c) no solution

(a) 1 sol'n



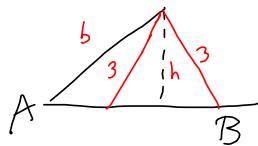
$$b \leq 3$$

(b) 2 solns:

$$\text{Need } b > a > h$$

$$\frac{h}{b} = \sin A$$

$$h = b \sin A$$



$$b > 3 > h$$

$$b > 3 > b \sin A$$

$$b \sin A < 3$$

$$b < \frac{3}{\sin A} = 3 \csc A$$

$$a \leq b$$

(c) No solns:

$$3 < h = b \sin 40^\circ$$

$$b \sin 40^\circ > 3$$

$$b > 3 \csc 40^\circ$$

$$3 < b < 3 \csc 40^\circ = 3 \csc 40^\circ$$

$$3 < b < 3 \csc 40^\circ$$