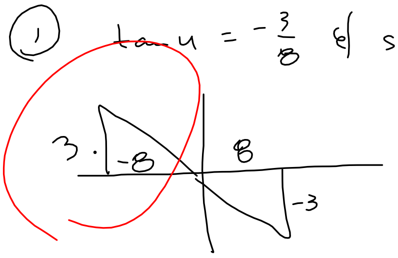
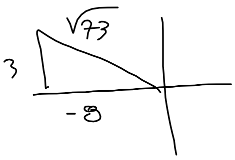
① $\tan u = -\frac{3}{8}$ & $\sin u > 0$




$8^2 + 3^2 = 73$




$\sin u = \frac{3}{\sqrt{73}}$ $\quad \csc u = \frac{\sqrt{73}}{3}$

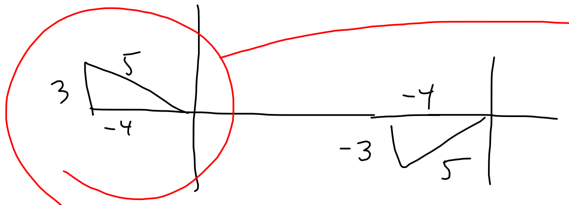
$\cos u = -\frac{8}{\sqrt{73}}$ $\quad \sec u = -\frac{\sqrt{73}}{8}$

$\tan u = -\frac{3}{8}$ $\quad \cot u = -\frac{8}{3}$

#1 was #2.

③ Find $\sin\left(\frac{u}{2}\right)$, $\cos\left(\frac{u}{2}\right)$, $\tan\left(\frac{u}{2}\right)$

given that $\cos(u) = -\frac{4}{5}$ & $\tan u < 0$




$\tan u < 0$

So, $\frac{\pi}{2} < u < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{u}{2} < \frac{\pi}{2}$

$\frac{u}{2} \in QI$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}} = +\sqrt{\frac{1-\cos u}{2}}$$

$$= \sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3}{\sqrt{10}} = \sin\left(\frac{u}{2}\right)}$$

QI

$$\cos\left(\frac{u}{2}\right) = \cancel{\pm} + \sqrt{\frac{1+\cos u}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{10}} = \boxed{\frac{1}{\sqrt{10}} = \cos\left(\frac{u}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{u}{2}\right) = \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{1} = \boxed{3 = \tan\left(\frac{u}{2}\right)}$$

(4) $4\cos^3(x) - 2\cos^2(x) - 2\cos(x) + 1$

$(2\cos^2 x)\underline{(2\cos x - 1)} - 1\,\underline{(2\cos x - 1)}$

$= (2\cos x - 1)(\underbrace{2\cos^2 x}_{(\sqrt{2}\cos x)^2} - 1) = (2\cos x - 1)(\sqrt{2}\cos x - 1)(\sqrt{2}\cos x + 1)$

$2\cos x - 1 = 0$

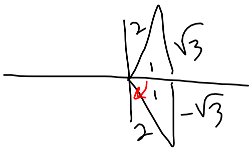
$2\cos x = 1$

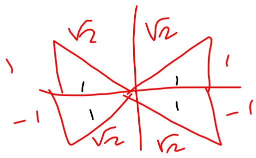
$\cos x = \frac{1}{2}$

$2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$



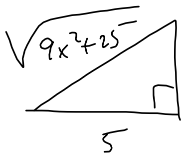




$x \in \left\{\frac{\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

(b) $\frac{\pi}{3} + 2n\pi$ OR $\frac{5\pi}{3} + 2n\pi$ OR $\frac{\pi}{4} + n\pi$ OR $\frac{3\pi}{4} + n\pi$,

$\forall\, n \in \mathbb{Z}$

(5) $\sin(\underbrace{\arccos\left(\frac{5}{\sqrt{9x^2+25}}\right)}_{\theta})$




$b = 3x$

$$\boxed{\sin(\theta) = \frac{3x}{\sqrt{9x^2+25}}}$$

$b = \sqrt{\left(\sqrt{9x^2+25}\right)^2 - 5^2}$

$= \sqrt{9x^2 + 25 - 25}$

$= \sqrt{9x^2} = |3x| = 3x$

Don't sweat the $|3x|$

6 $(\cos x - 1)^2 = (\sin x)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$\cos^2 x - 2\cos x + 1 = \sin^2 x$

$= 1 - \cos^2 x$

$+\cos^2 x \qquad +\cos^2 x$

$2\cos^2 x - 2\cos x = 0$

$(2\cos x)(\cos x - 1) = 0$

$\cos x = 0 \qquad \cos x = 1$

$x \in \{0, \cancel{\frac{\pi}{2}}, \frac{3\pi}{2}\}$

Check $\cos x - 1 = \sin x$

$\cos(0) - 1 = 0 \overset{?}{=} \sin(0) = 0$ ✓

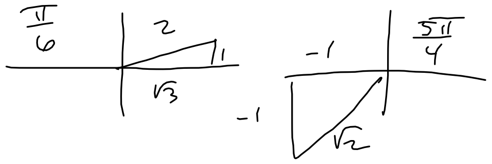
$\cos(\frac{\pi}{2}) - 1 = 0 - 1 = -1 \overset{?}{=} \sin(\frac{\pi}{2})$ No

$\boxed{x = \frac{\pi}{2} \text{ is extraneous}}$

You check $x = \frac{3\pi}{2}$.

7 a $\sin\left(\frac{17\pi}{12}\right) = \sin\left(\frac{2\pi}{12} + \frac{15\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{5\pi}{4}\right)$

$= \left(\sin\frac{\pi}{6}\right)\left(\cos\frac{5\pi}{4}\right) + \left(\sin\frac{5\pi}{4}\right)\left(\cos\frac{\pi}{6}\right)$

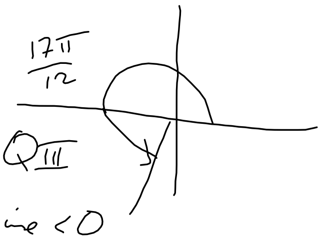



$\sin(u+v) =$
$(\sin u)(\cos v) + (\sin v)(\cos u)$

$= \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{-1-\sqrt{3}}{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

b Half-Angle ID: $\frac{u}{2} = \frac{17\pi}{12} = \frac{1}{2}\cdot\frac{17\pi}{6} = \frac{1}{2}\cdot u$

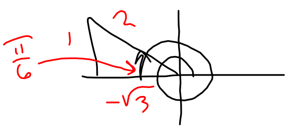



$\Rightarrow$ Q III

$\Rightarrow \sin < 0$

$\frac{17\pi}{12} = \frac{12\pi}{12} + \frac{5\pi}{12} = \pi + \frac{5\pi}{12}$

$\sin\left(\frac{\frac{17\pi}{6}}{2}\right) = -\sqrt{\frac{1-\cos\left(\frac{17\pi}{6}\right)}{2}}$



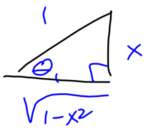

$= -\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$

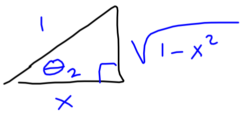
$= -\sqrt{\frac{2+\sqrt{3}}{4}}$

$$\boxed{\begin{aligned} &= -\frac{\sqrt{2+\sqrt{3}}}{2} \\ &= \sin\frac{17\pi}{12}\end{aligned}}$$

8) $\sin(\arcsin(x) + \arccos(x))$  $\sin(u+v) = \sin u \cos v + \sin v \cos u$

$= \sin(\overset{\theta_1}{\arcsin x})\cos(\overset{\theta_2}{\arccos x}) + \sin(\arccos x)\cos(\arcsin x)$




$\sqrt{x^2} = |x|$

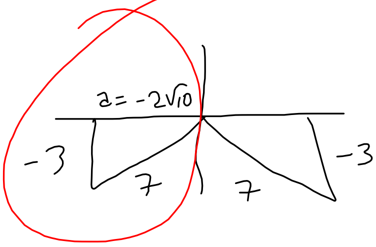
$(\sqrt{x})^2 = x$

$= (x)(x) + (\sqrt{1-x^2})(\sqrt{1-x^2})$

$= x^2 + 1 - x^2 = \boxed{1}$

9) $\sin u = -\frac{3}{7}$ & $\cos u < 0$  Find $\sin(2u)$, $\cos(2u)$, $\tan(2u)$

$\sqrt{7^2 - (-3)^2} = \sqrt{40} = 2\sqrt{10}$




$\sin(2u) = 2\sin u \cos u$

$= 2\left(-\frac{3}{7}\right)\left(-\frac{2\sqrt{10}}{7}\right) = \boxed{\frac{12\sqrt{10}}{49} = \sin(2u)}$

$\cos(2u) = 2\cos^2 u - 1$

$= 2\left(\frac{-2\sqrt{10}}{7}\right)^2 - 1$

$= 2\left(\frac{4(10)}{49}\right) - 1$

$= \frac{80-49}{49} = \boxed{\frac{31}{49} = \cos(2u)}$

$\tan(2u) = \frac{12\sqrt{10}}{49} \cdot \frac{49}{31}$

$= \boxed{\frac{12\sqrt{10}}{31} = \tan(2u)}$

10 arc length: $s = r\theta = 8\,(2100^\circ)\left(\frac{\pi}{180}\right)$

$= \frac{(\cancel{8}^{4})(\cancel{2100}^{70})\pi}{\cancel{180}_{\cancel{9}\,3}} = \frac{280\pi}{3}$

$\frac{(8)(2100)\pi}{180}$

$= \frac{2^3 \cdot 2^2 \cdot 3 \cdot 5^2 \cdot 7\,\pi}{2^2 \cdot 3^2 \cdot 5}$

$= \frac{280\pi}{3}$

$2 \,|\, 8$, $2 \,|\, 4$, $2$

$2 \,|\, 2100$, $2 \,|\, 1050$, $3 \,|\, 525$, $5 \,|\, 175$, $5 \,|\, 35$, $7$

$2 \,|\, 180$, $2 \,|\, 90$, $3 \,|\, 45$, $3 \,|\, 15$, $5$

B1

$87 \div 9 = 9 \text{ r}6$: $87 - 81 = 6$

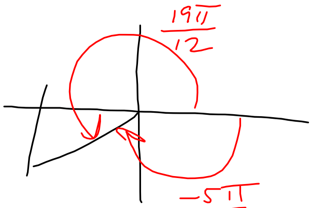
$\frac{91\pi}{12}$

$\frac{\frac{91\pi}{12}}{2\pi} = 3.\ldots$

$(3)(2\pi) = 6\pi$

$\frac{91\pi}{12} - 6\pi = \frac{(91-72)\pi}{12} = \frac{19\pi}{12}$

$\boxed{\frac{19\pi}{12},\ -\frac{5\pi}{12}}$




$2\pi - \frac{19\pi}{12}$

$= \frac{24\pi - 19\pi}{12}$

$= \frac{5\pi}{12}$

B1) Degrees Version

$$\left(\frac{91\pi}{12}\right)\left(\frac{180^\circ}{\pi}\right) = 1365^\circ$$

$$\frac{1365}{360} = 3 \text{ something}$$

$$1365 - (3)(360) = 285^\circ$$

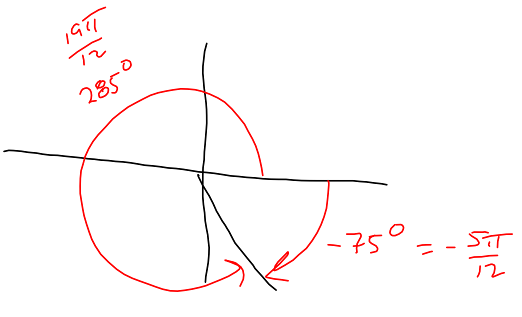
$$285^\circ = (285^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{19\pi}{12} \text{ rads}$$

$$360^\circ - 285^\circ = 75^\circ$$

$$(75^\circ)\left(\frac{\pi}{180}\right) = \frac{5\pi}{12}, \text{ so } -\frac{5\pi}{12}$$

OR, just

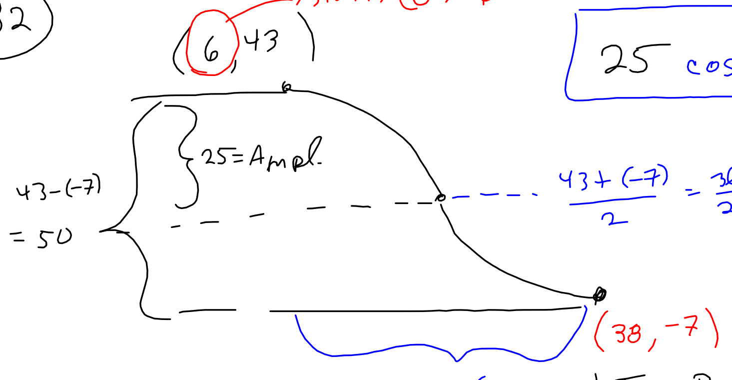
$$2\pi - \frac{19\pi}{12} = \frac{24\pi - 19\pi}{12} = \frac{5\pi}{12} \text{ so } -\frac{5\pi}{12}$$



B2

starts @ x=6

$$\boxed{25\cos\left(\frac{\pi}{32}(x-6)\right)+18}$$




$\frac{43+(-7)}{2} = \frac{36}{2} = 18$ = midline

$38-6=32=\frac{1}{2}T = \frac{\text{Period}}{2}$

So, $T = 64$

$bx = 2\pi$ when $x=64$

$64b = 2\pi$

$b = \frac{\pi}{32}$

B3

$$\text{AREA} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2}(50)^2(315^\circ)\left(\frac{\pi}{180^\circ}\right)$$

$$\frac{(2500)(315)\pi}{2(180)}$$

$$\frac{\cancel{2}^{2} \cdot 5^4 \cdot \cancel{3}^{2} \cdot \cancel{5} \cdot 7}{2 \cdot \cancel{2}^{2} \cdot \cancel{3}^{2} \cdot \cancel{5}} = \frac{5^4 \cdot 7}{2}$$

$$= \frac{625 \cdot 7}{2} = \frac{4375}{2}$$

2 | 2500, 2 | 1250, 5 | 625, 5 | 125, 5 | 25, 5

3 | 315, 3 | 105, 5 | 35, 7

2 | 180, 2 | 90, 3 | 45, 3 | 15, 5

So $\boxed{\frac{4375\pi}{2} \text{ cm}^2}$

B4